The Amicus Game*

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Abstract

Despite increased scholarly attention towards analyzing the influence of amicus briefs on case outcomes, we lack a microfounded model for understanding what we observe. Our analysis remedies this gap, modeling a world in which potential filers can advocate for a particular ruling and may provide information to influence a judge's decision. We show that the influence of an amicus brief depends upon the interaction of the group's bias and contextual factors. Specifically, while the influence of biased groups is sensitive to features of each case, such as the stakes of the issue, moderate group influence is relatively stable. Our findings are also relevant for empirical studies, they indicate that analyses of influence with observational data are likely undermined by a failure to account for strategic group behavior. Notably, analyzing only filed briefs will generate biased estimates of influence unless the researcher accounts for a group's interest in each particular case.

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1 Introduction

Amicus curiae briefs, although long-existing (e.g., Kochevar (2013)) and employed in a variety of contexts, emerged to great prominence in the second-half of the 20th century.[1] Notably, their numbers proliferated in the United States court system, especially in the Supreme Court (e.g., Owens and Epstein (2005); Salzman, Williams and Calvin (2011)).[2] For judicial decision-making, amicus briefs are considered fundamental for providing factual evidence to the courts, albeit "funneled through the screen of advocacy" (Larsen (2014), p. 1757; on the uniqueness of amicus information, see Collins Jr, Corley and Hamner (2014)). Friends of the court briefs may offer perspectives and data about how the world works and how a judicial ruling may impact society that would not be found in briefs authored by plaintiffs or defendants. While many issues are ideologically polarizing (e.g., Swenson (2016)), a good amicus brief is typically portrayed as valuable for providing distinct, hard, evidence to judges trying to make tough legal decisions rather than as a simple statement of general ideological leanings.[3]

How influential are these briefs for judicial decision-making, and what causes this influence to vary? Given the prominence of amicus briefs and their potential to inform the court, discerning the influence of briefs is crucial for understanding the amicus process and the role that outside groups play in shaping judicial outcomes. Accordingly, scholars have increasingly focused on the seeming impacts of amicus briefs and identifying the underlying processes at work. Descriptively, and not surprisingly given their rise in numbers, briefs are more likely than previously to be directly cited in opinions (e.g., Owens and Epstein (2005)) or to have their language lifted from them (e.g., Collins Jr, Corley and Hamner (2015); also see Sim, Routledge and Smith (2015)). In some instances, the sheer number of briefs, conditioned by the ideological position being advanced, has been found to be important for outcomes or getting cases on the Supreme Court docket, but—consistent with the notion that quality and credibility likely trump quantity—results have been inconsistent or asymmetric (e.g., Collins Jr, Corley and Hamner (2015); Hazelton, Hinkle and Spriggs (2017)). Finally,

[1] Various reasons offered for this development include a dramatic rise in the count of interest groups, the Supreme Court docket's subsequent reduction, a more liberal interpretation of Court rules allowing briefs from those other than the Solicitor General and state Attorneys General, and the emergence of a Supreme Court bar which both sells amicus briefs and employs them as advertising for firm quality (e.g., Ward (2007), Howard (2015), Larsen (2017)).

[2] Although there has been growth, compared to the changes at the Supreme Court the number of amicus briefs in the U.S. appeals courts remain rather modest (e.g., Martinek (2006), Gidiere III (2012)). The Supreme Court's use of amici is also very high relative to other nation-states (Collins Jr and McCarthy (2017)).

[3] Throughout our discussion we will assume that judges have lifetime tenure. The effects of briefs may be different if judges face reelection constraints (e.g., Becker Kane (2017)).

and related to the possibility that briefs need to be high quality and from well-regarded sources, others have emphasized the importance of networks collaborating together in their friendship efforts. Most notably, Box-Steffensmeier, Christenson and Hitt (2013) (see also Box-Steffensmeier and Christenson (2014)) conclude that better connected groups are more successful than others in influencing judicial behavior.

Despite all of these efforts and their corresponding insights, what has been lacking are clear theoretical foundations for analyzing amicus brief influence. "The theoretical motivation for these studies [of amicus briefs] are not as well developed as it could be," one recent review of the literature acknowledges (Perkins and Collins Jr (2017), p. 367). Our analysis addresses this gap by developing a game-theoretic model of the amicus process. We show that a brief's influence depends highly on the filing group's motivations. It also provides theoretically-based guidelines for improving empirical estimates of the influence of interest group briefs.

Hence, we offer the first attempt that we know of to build a microfounded theory of amicus behavior. In doing so, we need emphasize that our analysis focuses on briefs that are at least potentially informative to the judge. Distinctions between potentially viable filings and those that are mere dross with little chance of engaging judges (a critique of many briefs most associated with scholar and Judge Richard Posner; for discussions, see Lynch (2004); Garcia (2008)) have received considerable scholarly attention and inform our modeling choices (e.g., Zuber, Sommer and Parent (2015); Larsen and Devins (2016); Solimine (2016)).[4] In our model we capture this concept of potentially viable briefs by stipulating that possible purveyors of amicus briefs who might sway a pivotal judge each decide whether to pay the cost of producing a brief and whether to include hard, verifiable, information that might be useful to the judicial decision-maker. Thus, while the group can write a brief without verifiable information, the important aspect of our model from both a conceptual and a strategic standpoint is that the group at least has access to such information and has the option of integrating it into a brief. Subsequently, the judge may or may not thoroughly read the information contained in each brief and then decides the issue.

Importantly, we analyze a world in which the influence of such information is shaped by the ideological leanings of the groups and the receiving judge. Potential filers can either be biased and always prefer the same outcome, regardless of the case facts, or moderate, in which case the group's preferred decision is fact dependent. Similarly, the judge weights the technical merits of each side, but also may be ideologically biased in favor of a particular

[4]See Posner's decision in *Voices for Choices v. Ill. Bell Tel. Co.*, 339 F.3d 542 (7th Cir. 2003). We should note that we allow the attributes of potentially viable briefs to vary by analyzing the relative quality of the information to which groups have access.

decision. While the judge is not purely ideologically driven, varying the intensity of her ideological bias allows us to capture different case types. For instance, highly ideological issues would be those where most groups are biased and the judge weights personal policy preferences highly, while more technical issues may feature more moderate groups and the judge placing little weight on personal ideology.

As indicated, we present three sets of results utilizing our framework that furnish theoretical insights and speak to empirical research on amicus influence. First, we show that the influence of briefs filed by groups with strong biases is sensitive to variation in the value that these filers place on the case at hand as well as differences in the judge's likelihood of carefully reading their briefs. Conversely, the influence of relatively moderate groups is less sensitive to the value they place on the issue and, surprisingly, is not at all conditioned by variation in the judge's likelihood of reading their filings. This discrepancy highlights the importance of heterogeneity in the amicus process, as a group's influence may depend upon an interaction of contextual factors and ideological bias. In contrast to these ideologically-conditional effects of the stakes of the case to groups and judicial scrutiny, we show that the effects of variation in the quality of a group's information are similar for moderate and biased groups. Regardless of ideology, a group with low quality information has little influence.

Second, we draw out the implications of our theoretical results for empirical studies of amicus influence. Existing observational work on amicus influence must confront the fundamental problem of causal inference: it is not possible to observe the counterfactual outcome if a filing group had not filed, or if a non-filing group had instead filed. Our model provides a formal framework to think through these counterfactuals. Indeed, we connect the model's insights to the well-known Rubin causal model (Holland, 1986), deriving novel implications for empirical studies of amicus influence. Specifically, our model highlights a potentially key force that is unaccounted for in empirical studies of amicus behavior: that the *absence* of a filing by an interested group provides a judge with germane information but the failure to file by a group that does not care is uninformative. This is rooted in the strategic nature of information in our model. A judge knowing group characteristics, specifically whether or not it cares about the issue at hand, can learn from non-filing. In turn, our model pinpoints an important selection effect that is a function of of when groups file. Groups will only file if they have a stake in the case, but a non-filing may be generated by either a lack of interest or a prohibitively high filing cost. Thus, our model demonstrates formally how empirical analysis that fails to account for each case's relevance to each group will misestimate the influence of amicus briefs. In turn, we provide a blueprint for future empirical research centered on integrating a variable accounting for whether each case is relevant to each group being observed.

Finally, in light of our results on influence and heterogeneity in group bias, we discuss the welfare effects to judges of shifts in their attention to various (i.e., moderate or biased) groups. First, we show that judicial allocation of additional time and resources for analyzing moderate group briefs is inefficient. This is due to the strategic nature of information transmission that our results highlight. When moderate groups file, the information they provide is always truthful and verifiable by the judge. Given this, the judge need not verify the contents of moderate groups' briefs. In contrast, the welfare effects of increased judicial attention to biased groups are ambiguous. On one hand, it is better for the judge to allocate some attention to verifying the content of biased briefs than to simply ignore them. However, further increasing the scrutiny level of such briefs may reduce judicial welfare because such attention may incentivize biased groups to free-ride on one another, lowering the overall quality and amount of information provided by such briefs.

Overall, our analysis demonstrates that potential filers are strongly incentivized to consider a situation's context and what motivates judicial decision-makers and others who might file a brief. Therefore, the influence of amicus briefs on judicial decision-making is conditioned by these factors as well. Judges, on one hand are frequently hungry for valuable information and, on the other hand, are well-aware that those pushing briefs have private interests that might conflict with their own predilections and attitudes. Motivations, information, and strategic choice behavior must all be incorporated for meaningful analysis of amicus choices and effects. Absent such integration, making full sense of what is observed empirically is problematic.

Specifically, we model amicus briefs as a form of informational lobbying by groups to influence judicial decision-making. In our model, groups attempt to persuade the judge by furnishing "hard information," by which we mean information where disagreement over the substance is difficult post-verification (Ijiri, 1975). Unlike traditional models of persuasion through hard information (e.g., Milgrom (1981)), we assume that, while a brief may contain hard information, a filing does not automatically imply that the judge observes this information. Instead, with some probability the brief is reviewed and the information is verified and with some probability the judge only observes that the group files a brief.

This kind of informational environment has been studied previously in the political science literature across a variety of substantive contexts. However, several features make our analysis examining distinct channels of influence on judicial decision-making complementary rather than redundant.

For example, we adopt a messaging and informational technology similar to that employed by Austen-Smith and Wright (1992, 1994) in studying legislative lobbying by a pair of groups with opposing biases. However, there are several important differences between the two analyses. For one, we consider a richer group environment allowing for moderate groups and an arbitrary number of groups while Austen-Smith and Wright allow for endogenous information acquisition by the decision-maker and groups. For another, we focus on how information provision influences decision-making and highlight how this influence is difficult to detect with observational data, while they derive empirical expectations for why groups target certain decision-makers and not others.

Gailmard and Patty (2017), who analyze the impact of politicized judicial review on notice-and-comment rulemaking by agencies, represents another relevant analysis for our own. In both their model and ours, interest groups can provide hard information to the court. However, while Gailmard and Patty study how the court should optimally commit to reviewing policies, a key focus of our study is how groups strategically influence outcomes by affecting the judge's beliefs. Also, while Gailmard and Patty examine the case in which the group's bias is known, we include both biased and moderate groups. Finally, the Gailmard and Patty analysis is extended to agency rulemaking to illuminate the process through which cases arrive at the court; by contrast, we abstract away from case origination and, alternatively, investigate the strategic incentives at play when multiple groups with varying biases and informational qualities attempt to influence a judge.

Finally, the analysis in Beim (2017), by which the Supreme Court strategically uses multiple, possibly conflicting, lower court rulings to inform both its review decision and subsequent ruling, has features similar to ours. As in our model (and the others previously mentioned) hard information is provided, this time by advocates to the lower courts through a noisy process that filters up the judicial hierarchy through a process of strategic information transmission. Similar to our results, Beim finds both that strategic behavior induces learning even in the absence of review, and that the court's interpretation of information is conditioned upon the providing source's known bias. However, in Beim's analysis the higher court must choose among a menu of cases to review, only learning the relevant facts presented to the lower court after making its review decision. By contrast, we abstract away from lower court rulings and the decision to review and, instead, focus on the provision of relevant facts directly to the court deciding a particular case.

2 The Amicus Model

We now specify our amicus model in which strategic interests decide whether or not to file a brief designed to influence judicial decision-making.

In our amicus model the relevant actors are a judge and a set of N groups (in the real world such actors might not formally be groups, but entities such as law firms, loosely organized coalitions of legal experts, or corporations). Specifically, the judge makes a decision, $d \in \{d_0, d_1\}$, about an issue.[5] Each group potentially possesses the will and capacity to produce a high quality amicus brief designed to influence the judge's decision. Put differently, as in many proceedings, we assume that there are actors with vested interests besides the plaintiff or defendant with the option to be part of the process to try and affect judicial outputs.

Consistent with conventional beliefs about what kinds of cases might induce an amicus filing, we assume that there are relevant technical or legal arguments that remain unknown to the judge before the case proceeds. Let $\omega \in \{0, 1\}$ denote the decision supported by the case's technical and legal merits. As is standard in incomplete information models, we assume the actors have a common prior belief over the state; in particular, assume that $\omega = 1$ with probability q and $\omega = 0$ with probability $1 - q$. In general, we let the judge's belief that $\omega = 1$ at any stage of the game be $\mu_J \in [0, 1]$.

Our game begins after a case is placed on the docket.[6] In the initial decision stage each group observes a private signal about ω, denoted $s_i \in \{0, 1\}$. With probability $\pi_i > \max\{q, 1-q\}$ group i's signal is correct and with probability $1-\pi_i$ it is not. This assumption about π_i ensures that each group's signal is more accurate than the prior belief. This parameter captures heterogeneity in each group's expertise regarding the relevant issue area.

After observing its signal, each group privately observes its own cost of filing a brief. Formally, given that producing a meaningful brief is costly, we assume that filing a brief costs c_i. Put differently, we conceptualize briefs as costly efforts principally driven by a desire to influence outcomes (e.g., Collins Jr (2008)).[7] Each group's cost is distributed according to the cdf $G_i(c_i)$ with associated pdf $g_i(c_i)$ and full support on the interval $[0, C]$. As these

[5]Modeling the judge's decision as dichotomous is consistent with the approach taken in empirical work studying the influence of briefs, where a binary coded case decision or voting choice is the outcome of interest (e.g, Caldeira and Wright (1988), Kearney and Merrill (2000), Collins Jr (2008), Black and Boyd (2013), Box-Steffensmeier, Christenson and Hitt (2013); see Collins Jr, Corley and Hamner (2015) for an exception). Qualitatively similar results holds if instead we allow the judge to author nuanced opinions by choosing any alternative between d_0 and d_1 and specify group preferences in a manner that is similar to the dichotomous model.

[6]Although our model focuses on the court's final decision, it could provide a basis for understanding the influence of amicus briefs at the cert stage. Previous models studying the Supreme Court's decision to review a case abstract from the role of interests groups in this process (Cameron, Segal and Songer (2000), Lax (2003), Bustos and Jacobi (N.d.)). Future work should consider how features unique to the cert stage would affect the conclusions drawn here.

[7]Writing and filing costs of an amicus brief were estimated at approximately 50 thousand dollars in 2004 (Lynch (2004)), which would be approaching 70 thousand 2018 dollars. Beyond monetary costs, a brief filing requires investing time and, consequently, incurring significant opportunity costs. Although organizational maintenance, self-promotion, or other symbolic considerations could play a role in some situations for some types of brief producers (Hansford, 2004), producing a brief still involves some expenditure of time and/or resources and these idiosyncrasies can be captured by the group specific distributions over costs. Finally, while we focus on uncertainty over the cost of filing, our model is isomorphic to one where a group's filing cost is known but its valuation of the issue at stake is private.

costs may vary depending on the filer, we allow the cdf G_i to differ for each group, e.g., a law firm, a firm with in-house lawyers, and a grassroots interest group can have different capacities. Costs are private, as each group knows best its costs for brief writing and filing.

Subsequently, each group i chooses to file a brief in favor of decision d_0, file in favor of decision d_1, or not file. Denote group i's choice as $a_i \in \{f_0, f_1, n\}$. A group's filing choice may depend upon its signal, though we do not restrict this choice. If the group files it may choose to write a brief that, if read, reveals the group's signal to the judge. This corresponds to a brief that includes hard information relevant to the judge's decision, such as facts, data, or statistical analysis. On the other hand, the group may file a brief that lacks such factual information. A group is free to file in favor of either decision regardless of the signal that it receives. Thus, it is possible that a group will file in favor of one decision, but by not including unfavorable hard information lie by omission.[8] However, it cannot advocate a particular decision with its filing by manipulating the brief's factual content.[9]

After groups file, we advance to the judicial decision stage. Consistent with the demands on justices' (and their staffs') time, not all filed amicus briefs are read or thoroughly evaluated by the judge.[10] We capture judicial attention by assuming that if group i files then with probability p_i the judge reads the brief and learns the information contained within it. Conversely, with probability $1 - p_i$ the judge only observes that the group files and the decision the brief endorses but sees no information to which the group had access.

This set-up is consistent with different foci in the literature, one stressing a brief's factual content and another emphasizing the act of filing. Thus, a brief's filing serves as either a means to transmit hard information, with the judge learning the group's private information given that she audits the brief, or as a costly signal, so that the judge does not learn the group's information directly but draws inferences about it based on her understanding of

[8]For more on this, see Larsen (2014), who argues that, while some briefs include useful factual information, others are filed at the 11^{th} hour and include advocacy for a particular ruling but fail to provide useful factual information.

[9]In principle such information could be manipulated. However many groups, especially those repeatedly interacting with courts, care about their reputations with judges. As the relevant information can potentially be verified, discovery of information falsification or distortion could severely undermine a group's credibility. Knowing this should dissuade a group from manipulating the facts contained in its filing. Furthermore, modeling each group as having the capacity to provide hard information captures the substantive idea that briefs can provide useful information to judges.

[10]For example, Justice Ginsburg, in describing the task of her clerks, states: "Their job is to give me a road map through the case, and then I can read the briefs. They also tell me which of the green briefs (Amicus) I can skip" (Peppers and Ward, 2012, p. 395). While clerks may do their best in selecting briefs for the judge to read, they do not necessarily have the resources or expertise to select briefs perfectly. This suggests that the process through which briefs arrive on judges' desks can be noisy, and, in particular, the group faces some uncertainty as to whether its brief will be carefully read when deciding to file. However, each p_i may be different depending on the features of the corresponding case. For instance, the types of cases attracting few briefs would fall into the part of the parameter space where p_i is close to 1.

strategic behavior.

Next, the judge experiences a shock to her utility given by ϵ, where ϵ is drawn uniformly over $[-1, 1]$. This shock can represent any non-amicus related factors that affect the judge's considerations of the two decisions, e.g., group uncertainty over the judge's personal bias, or favorable public opinion regarding the case that makes her more prone to be supportive. The shock also effectively smooths the judge's decision, which facilitates a more tractable equilibrium analysis. It also renders equilibrium outcomes probabilistic, allowing for natural connections between our model and statistical analysis of amicus influence.

Finally, the judge rules by deciding between d_0 or d_1 and the game ends.

Having laid out the game's structure, we now specify the players' utilities. We assume that the judge cares about getting the choice right on legal or technical grounds. Thus, if the final decision matches the state then the judge gets a payoff of 1 and if it does not she gets a payoff of 0. Let $\mathbb{I}_{d=d_\omega}$ be an indicator function that takes a value of 1 if the decision matches the state and 0 otherwise. Consequently, the judge's payoff for choosing $d = d_1$ is $\mathbb{I}_{d_1=d_\omega} + \beta + \epsilon$, where $\beta \in (-1, 1)$ represents whether the judge has an ideological bias for or against decision d_1. Conversely, the judge's payoff for $d = d_0$ is simply $\mathbb{I}_{d_0=d_\omega}$.[11]

As for the groups, each group $i \in \{1, ..., N\}$ has known preferences over outcomes. Furthermore, groups can be categorized as either **biased** or **moderate**. If group i is biased in favor of decision d_1 then it gets a payoff of $v_i > 0$ when the judge chooses d_1 and a payoff of 0 otherwise. If group i is biased in favor of decision d_0 then its payoffs are specified analogously. Thus, the preferred decision of biased groups is invariant to the state of the world. On the other hand, if group i is moderate then its preferences over decisions depend on ω and it wants the judge's decision to match the state. As such, its payoff for decision d is $\mathbb{I}_{d=d_\omega} v_i$. Additionally, the v_i are common knowledge, with larger values representing greater stakes of the issue to group i.

To recap, our amicus brief game proceeds as follows:

1. The case is placed on the docket.

2. Nature determines which decision the legal and technical merits of the case favors, $\omega \in \{0, 1\}$.

3. Each group $i \in \{1, ..., N\}$ observes a private signal about ω, $s_i \in \{0, 1\}$, which is

[11] An alternative formulation of our model would specify multiple justices with different ideological biases toward each decision and who vote over the choices associated with d_0 or d_1. Under this setup, the median justice's vote is decisive. As previously shown, however, Supreme Court opinions may actually reflect the preferences of median of the majority coalition (e.g., Clark and Lauderdale (2010); Carrubba et al. (2012)). Thus, taking a reduced form approach and simply conceptualizing the single justice in our model as the median of the majority coalition may be better.

correct with probability π_i.

4. In the first stage of the process, each group decides whether or not to file an amicus brief and, if so, whether to include factual content in their filing.

5. If group i files then with probability p_i the judge learns the factual content (if any) of group i's brief and with probability $1 - p_i$ the judge only observes that the group filed.

6. In the final stage, the judge updates her beliefs and makes a decision, $d \in \{d_0, d_1\}$.

7. The judge receives a payoff of 1 if the decision matches the state and 0 otherwise; additionally, her utility is shifted by $\epsilon + \beta$ if she chooses $d = d_1$. Each group i gets utility v_i if the decision matches its preferred decision and 0 otherwise, and pays private costs c_i if it files a brief.

3 Equilibrium Analysis

Given that our amicus brief model features incomplete information, we analyze the game by studying a selection of perfect Bayesian equilibrium. As is standard in games with continuously distributed costs, we focus on equilibrium in which players use cut-point strategies. In such an equilibrium, each group uses a cut-off rule, conditioned on its signal, to determine whether to file, which decision to file in favor of, and whether to include hard information. In our case a cut-point equilibrium has three desirable properties (i) actions are in pure strategies, (ii) each group's decision is monotonic in its costs, and (iii) equilibrium behavior does not depend on strong assumptions about beliefs after off-path actions. In the following paragraphs, we define what a cut-point strategy entails, detailing behaviors of the judge, biased groups, and moderate groups. Then, we establish equilibrium existence using these strategies and discuss additional characterization. Proofs of all results can be found in the online Appendix.

We begin with optimal judicial decision-making. Consistent with legal scholarship studying use of amicus briefs (e.g., Larsen (2014)), the judge accounts for each group's bias and incentives to influence her decision and rationally updates her belief over the state. Define a profile of outcomes as $o = (o_1, ...o_N)$. Specifically, for each group i, $o_i \in \{n, f_0, f_1, 0, 1, \phi\}$ denotes if the judge observes that (i) the group did not file, (ii) the group filed for d_0 or d_1 but not the brief's content, (iii) the brief includes hard information supporting d_0 or d_1, and (iv) the brief contains no useful information. The judge's belief that $\omega = 1$ following outcome o is $\mu_J(o)$, and it is derived via Bayes' rule whenever possible. The judge maximizes her expected utility after updating her belief. Formally, if $\mu_J(o) + \beta + \epsilon \geq 1 - \mu_J(o)$ then $d = d_1$ and if $\mu_J(o) + \beta + \epsilon < 1 - \mu_J(o)$ then $d = d_0$.[12]

Next, we turn to each group's decision of whether or not to file. As the judge cannot observe a group's cost, she forms expectations about when the group files and adjusts her beliefs accordingly. Furthermore, with all groups making simultaneous filing decisions, each forms expectations about the others' filing behaviors. Thus, each group accounts for both the judge's and its own expectations about whether other groups will file. In a cut-point equilibrium we assume that group i's strategy is characterized by cut-points $\bar{c}_i(s_i) \geq 0$ such that, after observing signal s_i, if $c_i \leq \bar{c}_i(s_i)$ then the group files and if $c_i > \bar{c}_i(s_i)$ then it does not. Let $\hat{c}_{-i}$ be the conjectured set of cut-points used by groups other than i. Given these conjectures, the group files if after observing signal $s_i \in \{0, 1\}$ its expected utility for filing is greater than its expected utility for not filing. This results in a group filing following signal s_i if its cost is below a cut-point $\bar{c}_i(s_i)$.

Additionally, if a group files it does so in support of the policy it prefers, conditional on its observed signal. When using cut-point strategies a biased group always files in favor of its bias, and only includes the factual information contained in its signal if it is consistent with its bias. By contrast, a moderate group always files in favor of the state that matches its signal, and always includes hard information corresponding to its signal when it files.

Of course, while it may be optimal for a single group to use such a strategy given its expectations about the behavior of the other players, equilibrium requires that this optimality holds simultaneously for *every* group given their strategies. For this characterization, it is crucial that groups are expected by one another, and the judge, to file according to a cut-point strategy. Moreover, the group's filing strategy must be optimal given the judge's beliefs about the group's information. The following result establishes existence, a byproduct of which is a simple characterization of equilibrium cut-points.

Proposition 1. ***(Existence and characterization)***

1. *All perfect Bayesian equilibria of the game are in cut-point strategies. Furthermore, there exists a cut-point equilibrium in which every group files with positive probability.*

2. *Consider a cut-point equilibrium in which group i files with positive probability. If group i is biased in favor of decision d_1, then $\bar{c}_i(1) > \bar{c}_i(0) \geq 0$. By contrast, if group i is biased in favor of decision d_0 then $\bar{c}_i(0) > \bar{c}_i(1) \geq 0$.*

3. *For a given set of outcomes o there exists a unique $\bar{\epsilon}(\mu(o))$ such that if $\epsilon \geq \bar{\epsilon}(\mu(o))$ then the judge chooses $d = d_1$; otherwise, if $\epsilon < \bar{\epsilon}(\mu(o))$ then the judge chooses $d = d_0$.*

[12] We assume the judge rules in favor of d_1 when indifferent. This is inconsequential as, given the shock ϵ, it is a probability 0 event.

Importantly, in equilibrium the costs for which a group is willing to file or not is a function of its signal.[13] As Figure 1 shows, a biased group files more often when it has favorable information compared to when it has observed an unfavorable signal. This difference in filing arises because the possibility that the judge carefully reads the group's brief makes filing riskier for a biased group with unfavorable information. This discrepancy in cut-points enables the judge to infer information about a group's signal just from observing whether the group filed. As biased groups with favorable signals file more often, a judge only observing that the group files updates her beliefs favorably toward the group. Given biased groups' equilibrium behavior, if the judge reads a brief from a biased group that lacks hard supporting information then she correctly infers that the group's information is unfavorable. Consequently, the group is unwilling to incur high filing costs when it is unable to provide a brief with favorable, factual, information. On the other hand, with sufficiently low costs the group still sometimes files despite unfavorable information, hoping that the judge will not extract and learn the group's inability to provide favorable factual information.

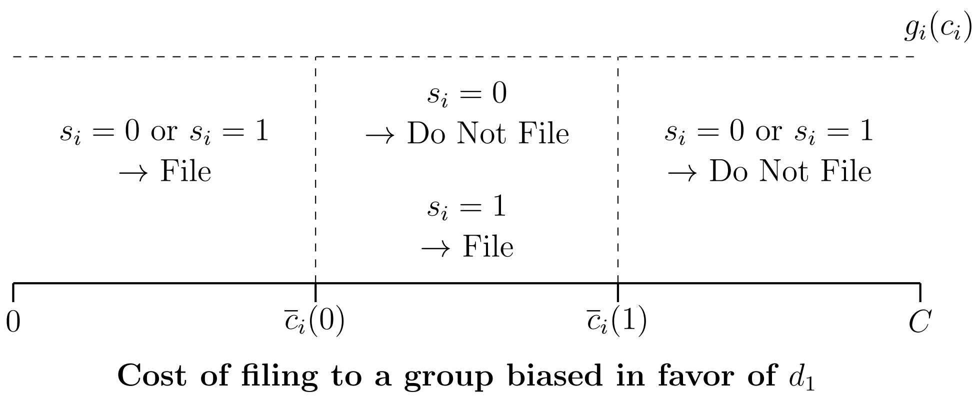

Figure 1: On the left side of the diagram, groups have very low filing costs and filing occurs no matter what signal is received; on the right side of the diagram, filing is prohibitively costly and filing never occurs regardless of signal; and in the shaded middle region, costs are at an intermediate level and groups file conditional upon observing a favorable signal.

Having examined the relationship between costs and bias, we now discuss how bias impacts informativeness. Cut-point equilibria also capture an important distinction between moderate and biased groups regarding filing informativeness. When moderate groups file briefs they support the decision that corresponds to their signals. The judge is thus able

[13] Even after restricting attention to equilibria with a cut-point form, multiple cut-point equilibrium may exist. This is due to different expectations over cut-points possibly leading to alternative solutions to the group's indifference conditions. In addition to the equilibria we focus on in which every group files with positive probability, there may exist "degenerate" cut-point equilibria in which one or more groups never file. However, such equilibria rely on unreasonable off-path beliefs in the filing stage.

to infer that these group have factual information that supports the decision endorsed by the briefs, without having to consider carefully the arguments provided in the brief. On the other hand, as briefs filed by biased groups support the same decision regardless of their factual information, the judge is unable to discern perfectly whether the brief provides hard information supporting the position advocated without going through and extracting the information in the brief. In this sense, moderate group briefs are more informative than those provided by biased groups.[14]

In equilibrium, groups account for the expected filing decisions of all other groups when making their own filing decisions. Thus, a group may be more or less likely to file, depending on its expectations about the filing strategies of both friendly and opposing groups. As the model allows for an arbitrary number of groups, in general this can result in quite complicated incentives. Cut-point equilibria account for such complexities, but it can be difficult to parse exactly how the presence of other groups influences incentives. To better highlight these effects, we now discuss two strategic tensions that can arise in a cut-point equilibrium due to the interaction of multiple groups. In the online Appendix, we analyze cut-point equilibria in special cases of our model that further explore these trade-offs.

The first such tension involves incentives for groups to free-ride on each others' filings. That is, a group may file less frequently if it anticipates that another group with similar preferences will file. This incentive can arise in cut-point equilibria, and we illustrate this in the Appendix using a special case of the model with two similarly biased groups. Because the incentive to free-ride is present in equilibrium, our model incorporates a strategic rationale for groups to try and overcome this impediment by coordinating on filings ahead of time. Indeed, the broader literature on amicus filings emphasizes that groups frequently coordinate with one another when filing. While we do not explicitly model this coordination process, an individual group in our model can be interpreted as reflecting the combined effort of multiple groups effectively overcoming the free-rider problem. In this case, assuming that the group has a high π_i may be reasonable due to it having the combined knowledge and resources of multiple allies. Thus, by treating the set of filers as fixed, our model incorporates pre-filing coordination to file jointly in a reduced form manner. A promising avenue for future work would be to model explicitly how and when groups coordinate ahead of time to file jointly.

The second strategic tension concerns how groups respond to the existence of other groups with opposing preferences. In equilibrium, a group's filing decision must also consider the

[14] Technically, there could exist cut-point equilibria in which the biased groups "babble" by mixing over which side they support or by always filing in favor of their least preferred decision. However, in all these equilibria the judge never changes her belief about a biased group's signal based solely on the side for which it states support. As incorporating babbling has no effect on equilibrium outcomes, our results and discussion refer to the intuitive equilibria in which biased groups endorse their preferred decision.

filing decisions of opposing groups. This can have consequences for not only filing behavior, but judicial welfare as well. In some cases, competition can improve the judge's welfare. This is because each group's filing behavior becomes more disciplined and this improves the judge's ability to infer information from filings. In other cases, however, adding a second competing group may lower judicial welfare by decreasing the informativeness of behavior when neither group files. We provide specific illustrations of each of these possibilities in the Appendix.

4 The Influence of Amicus Briefs

We now turn to the actual influence of amicus briefs. As discussed, a central goal in the study of amicus briefs is determining whether or not briefs influence decision-making, especially on the Supreme Court. Fortunately, our formal framework allows a precise analysis of influence. We can derive clear results tying group interests and characteristics to equilibrium influence levels.

We follow the existing literature by defining influence as the difference in the probabilities that the judge rules in favor of a group's preferred outcome given that the group does and does not file. Formally, **influence** is given by

$$Pr(d = \delta | \text{ i files in favor of } d_j) - Pr(d = \delta | \text{ i does not file}),$$

where δ is the group's preferred decision after observing its signal. The first term in our definition of influence is the probability the group's preferred decision is implemented when when it files. However, as our definition of influence implies, this alone does not measure group influence, as we want to determine the impact filing a brief has on changing the outcome. We must account for the probability that the group would have gotten its preferred outcome had it not filed. This is captured by the second term. Overall, then, a brief's influence determines how much more likely the group is to get what it wants when it files versus when it does not.

For briefs to be influential they must provide judges with relevant information. Given that groups are free to file briefs whatever signal they receive, it is tempting to infer that briefs are unlikely to convey useful information. However, this is not the case. As implied by our next result, in equilibrium the judge will infer information from a group's brief that, in expectation, makes her more likely to side with the group.

Lemma 1. *For each group i if $p_i > 0$ and $v_i > 0$ then influence is strictly positive.*

Even though group bias is fully known and judges may fail to analyze carefully the information in briefs, the strategic nature of filing decisions means that they may still convey useful information about the state of the world. While the judge is fully aware of the bias of groups, and acknowledges that briefs are filed with a particular goal in mind (Larsen, 2014), she still finds value in briefs. The sheer act of filing a costly brief indicates that a group is more likely than not to possess information that is both useful and supports its preferred decision.

4.1 The Effect of Group Bias on Influence

Having established that briefs convey information, we turn to what causes variation in group influence. Because the judge does not always carefully read each brief, she infers information in these filings from the group's strategic behavior and, thus, information transmission is conditioned by known characteristics of the amici filers themselves. Accordingly, our analysis demonstrates that group identity and characteristics matter for understanding the influence of briefs on judicial decision-making.[15]

Perhaps the most notable difference among organizations likely to be sufficiently resourced to file a viable brief are their ideologies or biases. Intuitively, this should condition how judges view what such groups submit and, thus, how much influence any group's brief has on the outcome. But how differences in bias translate into differences in influence is not straightforward.

In this vein, our results show that biased and moderate groups differ fundamentally in the comparative stability of their influence with respect to some factors but not others. In particular, relative to moderate groups, biased groups' influence is more sensitive to variation in either issue stakes or the probability that a judge carefully reads a filing. By contrast, the influence of both biased and moderate groups is highly sensitive to the quality of the group's information, i.e., we find no discrepancy between how information quality conditions the impact of either type of group.

Our next proposition highlights that, indeed, there is a clear difference in whether a filer is moderate or biased on how much impact a filing has on the judge's belief.

[15] A related claim has been articulated by the "affected groups" hypothesis (Collins, 2004). This theory emphasizes that amicus briefs influence the court because they signal the breadth of interests that are potentially affected by the decision. Our model provides an alternative strategic logic for why filers' identities may contain relevant information — even though the judge's utility is not directly dependent upon these groups' preferences. As the germane information in filings involves groups' private signals, the judge uses the information that she has about groups' characteristics to form beliefs about the information that they possess, even without carefully reading their briefs.

Proposition 2. *Given judicial belief μ_J, after observing group i file in favor of decision d_1 the judge's updated belief is weakly greater when group i is moderate than when it is biased in favor of d_1. An analogous result holds for decision d_0.*

Proposition 2 implies that in the one group case influence is weakly larger for a moderate group. This is true even accounting for the fact that judges might read and determine a brief's factual content. It is also true in the N group case if we compare the judge's belief following the outcome of filings by each group besides group i with that after observing group i's filing. Put simply, filings by moderate groups are more informative than those by biased groups.

This result has important implications for empirical studies of amicus brief influence. When conducting such analyses, it is tempting to draw conclusions based only on observed filings. However, our results suggest that when analyzing the first term of influence of amicus briefs only counting such filings will bias results in favor of concluding that moderate groups are most influential. As the definition of influence makes clear, it is necessary to account for a judge's decision conditional on a group *not filing*. Indeed, it is possible that looking only at cases of "success" in the context of observed filings will suggest an incorrect pattern of influence.

To illustrate this potential for incorrectly attributing influence, consider the following simple example applying our model.

Example 1. *Assume there is only one group, the group knows the state with certainty, $\pi = 1$, the group places value $v = .9$ on the issue, costs for filing are drawn uniformly over $[0, 1]$, the prior belief is $q = .5$, and when the group files the judge observes the group's information with probability $p = .5$. In this example there is a unique cut-point equilibrium.*

If the group is moderate then $\bar{c}(1) = \bar{c}(0) = .45$. Given the group files, the probability it gets what it wants is 1. This is because the group knows the state and the judge knows that the group files in favor of its information. If it does not file then, since the group files with the identical probability following either signal, the judge does not update against the group and the probability it gets its preferred decision is .5. Thus, its influence when moderate is .5.

On the other hand, if the group is biased towards d_1, then $\bar{c}(1) \approx .54$ and $\bar{c}(0) \approx .085$. In this case, the probability that the group gets what it wants if it files is only .864. Thus, only observing the group when it files would lead one to conclude that the moderate group is more influential. However, a closer look reveals that the biased group only gets its preferred decision with probability .34 when it does not file. Thus, the influence of the biased group is .524, which is actually greater than that of the moderate group — even though in equilibrium the moderate group always achieves its preferred outcome when it files.

While Proposition 2 shows that the first term of influence is always higher for moderate groups, also note that this term is not a function of v for moderate groups. Thus, the probability that a moderate group gets what it wants when it files is *not* a function of how much it cares about the issue. This is not true, however, for biased groups for whom in general both terms depend on v. To highlight this difference, consider the following stark example where changing v does not affect moderate group influence but alters biased group influence.

Example 2. *Consider the parameters from example 1: $p = .5, q = .5$, $\pi = 1$, and costs are uniform, but let v vary. In this case, there exists a unique cut-point equilibrium.*

If the group is moderate, then how frequently it files changes depends on v. However, its influence remains .5 as, regardless of how frequently it files, a moderate group still obtains its preferred outcome when it files. Thus, influence is not a function of v. On the other hand, if the group is biased, then the influence of its amicus brief depends on its stakes in the issue. For example, when $v = .25$ its influence is approximately .3, while from the earlier example we know that raising v to .9 increases its influence to approximately .54.

Beyond differences in the value placed on the issue, the likelihood that the judge reads the information contained in a brief also varies by group. Similar to the results for variation in issue stakes, how sensitive a group's influence is to variation in the judge's probability of reading a brief is conditioned by whether the group is moderate or biased.

Proposition 3. ***(Influence and p_i)***

1. *If group i is biased then changing p_i affects its influence. In particular, if $p_i \to 0$ then group i's influence goes to 0. If p_i is sufficiently high then group i only files when its signal matches its preferred policy.*

2. *If group i is moderate then changing p_i does not change its influence.*

Proposition 3 demonstrates that the probability the judge extracts information from the contents of a brief significantly impacts the influence of biased group's filing decision and influence. A biased group's decision whether to file depends upon its observed signal. In the limit, it adopts the same filing decision rule whatever signal it receives, which results in it having no influence. By contrast, when the judge is very likely to read the brief's factual content, a biased group is very unlikely to file a brief without observing a favorable signal. Alternatively, a moderate group's influence is invariant to the probability that the judge reads the brief, as the decision that the moderate group files in favor for matches its signal.

Whether the judge reads its brief is inconsequential.[16]

Proposition 3 also demonstrates two benefits of policies strengthening judicial ability to assess claims. First, such policies could increase judicial expertise directly in any number of ways so that more information could be garnered from a filing. For example, judges could be allocated greater resources to hire high quality staff. Alternatively, rules could be changed so that briefs are required to be more transparent, which would strengthen de facto judicial expertise; for instance, consistent with the advocacy in Larsen (2014) for adopting measures increasing the methodological transparency of filings containing data as a means of reducing the ambiguity of the information contained in many briefs, standards for what constitute amicus "facts" could be implemented. Second, and less directly, policies strengthening judicial abilities to deal with claims could impact who files in a way providing the judge with more information. Specifically, greater judicial ability to assess briefs would discourage groups receiving unfavorable signals from filing in the first place. However, as we discuss in Section 4.3, due to overall equilibrium effects on group filing behavior there may be costs associated with such policies.

A final result from our analysis involves the quality of group information, the probability that the group's signal matches the state of the world, π_i. This result stands in contrast to the results for issue stakes or the probability that a brief is read. As discussed, moderate group influence is relatively invariant to changes in these latter factors. Conversely, as the following proposition shows, in some sense information quality works analogously regardless of group preferences.

Proposition 4. ***(Influence and*** π_i***)*** *If* $\pi_i \to \frac{1}{2}$ *then group* i *has no influence on the judge's decision, regardless of its bias.*

Some groups may have better access to high quality information relevant for judicial decision-makers than others. In our model, this translates into differences in π_i. Proposition 4 shows that low quality groups, where it is highly unlikely that the groups' signals match the state of the world, have little ability to change the judge's belief about the correct decision. Thus, unlike p_i, the quality of the group's signal affects the influence of both moderate and biased groups. Furthermore, unlike the stakes, v_i, signal quality affects both influence terms for moderate and biased groups. Consequently, a group's information quality is a strong determinant of its influence regardless of its ideology. One implication of this result is that

[16]Given our assumption of a binary state space, this implies that the judge perfectly infers the moderate group's signal when observing a filing. If we instead assumed a continuous state space, the judge would still infer that the moderate group's signal was favorable, though she would be unable to pinpoint the state exactly without reading the brief. As such, the difference between moderate and biased groups would continue to hold, although less starkly than with a binary space.

we should only observe filings from groups with some ability to provide the judge with useful information on a case's technical or legal elements.

In sum, our results suggest that the influence of biased groups is highly sensitive on their specific characteristics and the amicus environment. The influence of moderate groups, however, is much less context sensitive. Only for information quality do we find comparable results regardless of the group's position.

4.2 Empirically Assessing Amicus Influence

As discussed, measuring the influence of briefs on judicial decisions is the primary objective in many empirical studies of the amicus process. Furthermore, numerous attempts to assess the impact of amicus activity on judicial decisions empirically have produced mixed results. As Kearney and Merrill (2000) point out, empirical findings have been "confusing and contradictory" (p. 774), a description that remains apt to the present. Some uncover evidence indicating influence (Collins, 2004; Collins Jr, 2008; Becker Kane, 2017), others find no effect (Songer and Sheehan, 1993; Spriggs and Wahlbeck, 1997), and still others discover conditional impacts (Kearney and Merrill, 2000; Box-Steffensmeier and Christenson, 2014; Syzmer and Ginn, 2014). Our theory suggests potential reasons for these conflicting findings.

First, our formal definition of influence underscores the inherently *counterfactual* logic that arises in assessing how the amicus process may impact outcomes. Indeed, this definition is essentially the "average treatment effect" of a filing in the Rubin causal model, restated in terms of our formal model of amicus filings. Connecting our theoretical model to Rubin's framework concretely demonstrates the shortcomings of estimating influence based on observed filings and decisions.

Beyond applying insights from Rubin, our model points towards a subtle, related, issue due to equilibrium behavior: it is crucial to distinguish why a group does not file. As a preliminary, in Lemma 2 we distinguish the importance of relevance for a group; subsequently, we demonstrate how relevance impacts our ability to obtain an accurate estimate of amicus brief influence.

Lemma 2. ***(Influence and relevance)***

1. *If the case is relevant to group i, $v_i > 0$, then its equilibrium influence is strictly positive.*

2. *If the case is irrelevant to group i, $v_i = 0$, then its equilibrium influence is* 0.

Lemma 2 implies that a group is only expected to file with some probability if the case is relevant to its interests.[17] Consequently, a judge observing a non-filing by a group valuing the issue at hand believes that there is some probability the group received a signal contrary to its preferred policy. This implies that when a group values the issue at stake the judge learns from a non-filing.[18] By contrast, a group having no stake in the case is not expected to file. As a result, the judge does not change her beliefs given a non-filing by a group for which the case is irrelevant.

Thus, an accurate estimate of amicus brief influence requires treating non-filings by a group on relevant cases differently from non-filings on irrelevant cases. This difference highlights an important source of sample selection that, left unaccounted for, results in inaccurate group influence estimates. Our next proposition establishes this formally.

Proposition 5. *Let $\hat{I}$ be the observed difference in the probability a judge rules in favor of a group's preferred outcome among cases where a group files and where they do not file. $\hat{I}$ is strictly less than the true level of equilibrium influence, averaged across all cases.*

Proposition 5 demonstrates that failing to account for heterogeneity in group interest across observed cases produces a biased influence estimate. In particular, failing to account for group interest will bias findings *against* detecting influence. This stems from observed filings only coming from groups with a stake in the issue at hand, and observed non-filings arising either when a group is uninterested in the case or when a group is interested but finds the cost of filing prohibitive — which is particularly important if the group is unable to provide favorable information. While our theoretical analysis poses a serious challenge to existing empirical studies of amicus influence, leading us to question some of the findings, it also suggests a relatively simple solution by determining issue relevance to each group.

To make this more concrete, consider this problem in the context of several studies of amicus influence. Both Songer and Sheehan (1993) and Kearney and Merrill (2000) estimate amicus influence by comparing rates of "success" for petitioners and respondents when amicus briefs are and are not present. Our model suggests that this approach is likely to underestimate the impact of filings on these success rates, as cases may have no or few observed filings due to either a lack of interested filers or strategic concealment by interested filers.[19] Influence is only possible in the second circumstance, and pooling both reasons for

[17] Although distinguishing between groups with arbitrarily small v_i and groups with $v_i = 0$ may seem stark, this apparent abruptness disappears if there is a known fixed cost of filing. In this case, there is a strictly positive cut-off in v_i above which the issue is relevant for the group and below which it is irrelevant.

[18] This finding would not seem to be just an artifact of our model, as court observers note non-filings by important and relevant interests. For examples, see Kontorovich (2014) and O'Neil (2015).

[19] Both our results and these studies consider estimates of influence produced by taking a difference in the mean "success rate." An alternative approach would be to regress "success" on a group's decision to file. Our qualitative results carry over to this case, as estimates of the impact of filing will be biased unless one controls for group interest.

non-filing leads to an overestimate of the rate of success in the absence of observed filings. As Proposition 5 indicates, this results in a bias against detecting influence. This provides a logic to explain the mixed results for influence found in these studies, with Songer and Sheehan (1993) finding no evidence of influence and Kearney and Merrill (2000) finding no evidence for petitioner success, but some evidence of influence in the subset of institutional litigants such as the ACLU.

Although the issue that we have highlighted is fundamentally one of sample selection (i.e., do we include or exclude cases about which a group has no interest), our proposed solution is simpler than model-based sample selection corrections typically employed by social scientists. Here, a fix only requires coding a single additional variable for each potential filer indicating whether or not they value a case's outcome. With this variable in hand, bias can be avoided by first computing the difference in the probability the judge rules in favor of a group when it does and does not file among cases in which the group is interested in the outcome and then multiplying this by the proportion of observed cases in which the group has interest. According to our theoretical framework, this procedure provides an unbiased estimate of average influence.

4.3 Welfare Implications of Amicus Briefs

Given that briefs are influential for decision-making, the strategic filing of amicus briefs has significant welfare implications for the judge. Consequently, judges may be incentivized to direct their clerks to allocate special attention to briefs filed by groups with particular biases or known abilities to provide useful information (Peppers and Ward, 2012, p.395). We address this question within our formal model by considering how changes in p_i affect judicial welfare.[20]

First, notice that allocating resources towards filings of moderate groups is inefficient. Recall that, by Proposition 3, the value of p_i has no impact on the influence of moderate groups in equilibrium. This is because moderate groups truthfully file based on their signal, and so the group's behavior does not change whether or not it anticipates the brief will be vetted. Thus, the judge receives no benefit from having her clerks spend time closely reading moderate group filings, as they need only look at which side the group filing favors.

As implied, while allocating attention to moderate groups has no effect on the expected outcome, there may be an incentive to allocate resources to reading the briefs of biased

[20]In the Appendix we analyze special cases of our model in which the judge strategically chooses how much attention to pay to different filings after the briefs are filed. Specifically, we show that similar findings with regard to group bias hold.

groups.[21] To see this, recall the first component of proposition 3, which indicates that a biased group for which $p_i = 0$ has no influence on the judge's decision. In contrast, if $p_i > 0$, its filing will have some influence on judicial decision-making. This increased informativeness implies that there can be reason to direct clerks to allocate some time to reading the briefs of biased groups carefully.

However, while the judge may benefit from the increased informativeness resulting from increasing the attention paid to filings from biased groups, the overall welfare effect of this increased scrutiny is ambiguous. Indeed, it is possible that increasing p_i for a group decreases judicial welfare. In the Appendix we consider an example comparing a case with two biased groups, and show that judicial welfare can be higher in the case when $p_i = 0$ for one of the groups than in the case when $p_i > 0$ for both groups. In particular, the probability that the judge gets the state correct can be higher when she only allocates attention to a single biased group. The reason for this is that allocating attention to multiple groups induces an incentive to free-ride among groups with similar bias, which can reduce the overall informativeness of filings.

5 Conclusion

Amicus briefs have risen greatly in prominence over the last decades. At least in the American context, such briefs not only allow various interested parties to comment but they bring types of arguments and data that are typically not part of the case mounted by plaintiffs or defendants.

However, although legal scholars have spent much time and effort discussing what has changed and what makes a successful brief, and empirical scholars have amassed and analyzed a great deal of data from a variety of perspectives, we have lacked a microfoundation of the underlying process. While our model and its extensions are necessarily highly stylized, they offer a variety of insights into the roles of briefs and judges, and also provide a lens to understand empirical studies, the difficulties that they must confront, and what might constitute a solution.

We have shown that filer interests play an important role in conditioning influence. While the most preferred policies of groups are known in our model, amicus filers may vary both

[21] Building on what was previously discussed, this contrast between moderate and biased groups would be less stark with a continuous state space, however, the judge would still have more incentive to examine closely briefs filed by biased rather than moderate groups. With a continuous space, if the judge can only ascertain which side a group favors without reading the brief then, while a moderate group's filing is informative, the judge will be incentivized to read the brief to pin down the state precisely. Still, the judge would have a greater incentive to read a biased group's brief, as it is uninformative without closer examination.

in the value that they place on the issue at stake and in the quality of information that they can provide. Both characteristics interact with ideology to impact the decision to file and the amount of communicated information. However, both biased and moderate groups with poor information are unlikely to file and thus carry little influence. Neither the technical nor the strategic informational content of their filings are of use to the judiciary.

Our results also highlight previously unacknowledged roadblocks for empirically studying amicus influence with observational data. In particular, our model's informational logic demonstrates that the decision *not* to file a brief provides judges with useful information. We must distinguish between non-filings involving groups that are interested but not filing due to either prohibitive cost or the inability to provide useful information, and non-filings due to group apathy about case outcome. Not accounting for non-filing and these different mechanisms will result in faulty inferences about amicus influence's existence and degree.

Finally, we demonstrated that shifts in judicial attention to various groups can impact judicial welfare. These welfare results further reinforce the importance of heterogeneity in the amicus process. Increased attention to moderate groups has no effect on the outcome, and is consequently inefficient. In contrast, increased attention to biased groups has the potential to improve judicial welfare.

Since our analysis is the first to explicitly model amicus brief influence at the merits stage, there are a number of ways to build on it that leap to mind. For instance, our model begins after a case is placed on the docket. While a useful abstraction, future work may wish to analyze the impact of the cert or appeals process on the interaction that follows. Another interesting possibility would be to model explicitly dynamics of the amicus process; while our analysis takes group quality as given, some groups (e.g., law firms wishing to develop a robust Supreme Court practice) may interact repeatedly with the court and wish to establish a reputation for providing useful information. Another possibility would be to delve more deeply into coordination. Our analysis demonstrates that groups have an incentive to free-ride on the efforts of groups with similar interests and, thus, may want to coordinate with one another. Though we abstract from this by assuming that any coordination between groups has already occurred, explicitly modeling the process through which groups coordinate on filings may uncover interesting new incentives for groups that participate in the amicus process.

APPENDIX

Equilibrium Analysis

Proof of Proposition 1

We derive existence of cut-point equilibrium via a fixed point theorem. The crux of the proof is verifying continuity of a certain mapping representing each group's indifference condition between filing and not filing.

Recall that, given belief μ_J, the judge's expected utility for choosing d_1 is $\mu_J + \beta + \epsilon$ and her expected utility for d_0 is $1 - \mu_J$. Thus, we obtain the characterization of judicial behavior by setting $\overline{\epsilon} = 1 - 2\mu_J - \beta$. Additionally, note that $\overline{\epsilon}$ is strictly decreasing in μ_J.

Next, assume groups use cut-point strategies when deciding whether to file or not. Let $\tilde{o}$ be a realization of outcomes and $\tilde{o}_i$ be the outcome for group i in realization $\tilde{o}$. In particular, for each group i after observing signal s_i there is some cut-point $\overline{c}(s_i)$ such that if $c_i \leq \overline{c}(s_i)$ then the group files, otherwise it does not. Additionally, if group i is biased it always files in favor of its preferred decision while if it is moderate it files in favor of its signal.

Let $\mathbb{I}(\tilde{o}_i = y)$ be an indicator function which takes a value of 1 if $\tilde{o}_i = y$ and 0 for $\tilde{o}_i \neq y$. Given belief μ_J and conjectured cut-points $\hat{c} = (\hat{c}_1(0), \hat{c}_1(1), ...\hat{c}_N(0), \hat{c}_N(1))$, the probability that $\omega = 1$ after observing any outcome $\tilde{o}_i$ from group i is

$$
\begin{aligned}
&P(\omega = 1|\tilde{o}_i) = \\
&\qquad \mathbb{I}(\tilde{o}_i = 1)P(\omega = 1|\tilde{o}_i = 1) + \mathbb{I}(\tilde{o}_i = 0)P(\omega = 1|\tilde{o}_i = 0) \\
&\qquad + \mathbb{I}(\tilde{o}_i = f)P(\omega = 1|\tilde{o}_i = f) + \mathbb{I}(\tilde{o}_i = n)P(\omega = 1|\tilde{o}_i = n).
\end{aligned}
$$

Examining further, we can write each term as

$$
\begin{aligned}
&P(\omega = 1|\tilde{o}_i = 1) = \frac{\pi_i \mu_J}{\pi_i \mu_J + (1-\pi_i)(1-\mu_J)}, \\
&P(\omega = 1|\tilde{o}_i = 0) = \frac{\pi_i (1-\mu_J)}{\pi_i (1-\mu_J) + (1-\pi_i)\mu_J}, \\
&P(\omega = 1|\tilde{o}_i = f) = \\
&\frac{\mu_J\Big(\pi_i G(\hat{c}_i(1)) + (1-\pi_i)G(\hat{c}_i(0))\Big)}{\mu_J\Big(\pi_i G(\hat{c}_i(1)) + (1-\pi_i)G(\hat{c}_i(0)\Big) + (1-\mu_J)\Big(\pi_i G(\hat{c}_i(0)) + (1-\pi_i)G(\hat{c}_i(1)\Big)}, \\
&P(\omega = 1|\tilde{o}_i = n) = \\
&\frac{\mu_J\Big(\pi_i(1-G(\hat{c}_i(1))) + (1-\pi_i)(1-G(\hat{c}_i(0)))\Big)}{\mu_J\Big(\pi_i(1-G(\hat{c}_i(1))) + (1-\pi_i)(1-G(\hat{c}_i(0)))\Big) + (1-\mu_J)\Big(\pi_i(1-G(\hat{c}_i(0))) + (1-\pi_i)(1-G(\hat{c}_i(1)))\Big)}.
\end{aligned}
$$

The judge's belief following outcome $\tilde{o}$ is $\mu_J(\tilde{o}) = P(\omega = 1|\tilde{o})$. As groups' signals are independent conditional on ω we can obtain $\mu_J(\tilde{o})$ by using the individual $P(\omega = 1|\tilde{o}_i)$ and sequentially updating, replacing μ_J, initially q, with the new belief after each update. Thus, because G_i is continuous in c_i the $P(\omega = 1|\tilde{o}_i)$ are continuous in $\hat{c}_i$, and the resulting sequentially updated belief $\mu_J(\tilde{o})$ is continuous in $\hat{c}_i$.

Let $P(d = \delta_i|\mu_J(o_i, o^{-i}))$ be the probability that the judge's decision matches group i's preferred decision, given group i's outcome is o_i and o^{-i} is the $N-1$ tuple of outcomes for the other groups. Using our analysis of the judge's behavior, if $\delta_i = d_1$ this is the probability that $\epsilon > \bar{\epsilon}_{\mu(o_i, o^{-i})}$, i.e., $1 - F(\bar{\epsilon}_{\mu(o_i, o^{-i})})$.[22] On the other hand, if $\delta_i = d_0$ this is $F(\bar{\epsilon}_{\mu(o_i, o^{-i})})$. As $\mu_J(o)$ is continuous in $\hat{c}$ and $\bar{\epsilon}_\mu$ is continuous in μ we have that $P(d|\mu(o_i, o^{-i}))$ is continuous in $\hat{c}$. After group i observes signal s_i, using its expectation that other groups are using cut-point strategies $P(o^{-i}|s_i)$ represents the probability of outcome o^{-i} given signal s_i. Again, because the distribution over costs is continuous this yields that $P(o^{-i}|s_i)$ is continuous in $\hat{c}$.

For the next step, we introduce some notation. Let the probability that the judge implements decision d_1, after observing player i offer filing f_j, and given outcome $o \in \mathcal{O}_{-i}$ in strategy profile σ as

$$P_o^\sigma(f_j) = \Big[p_i P\Big(d = d_1|\mu(s_i, o)\Big) + (1-p_i)P\Big(d = d_1|\mu(f_j, o)\Big)\Big].$$

Additionally, for any $d \in \{d_0, d_1\}$ and $\omega \in \{0, 1\}$ define the following functions $V_i^g(d|\omega)$,

[22] Throughout, let F be the uniform distribution over $[-1, 1]$.

for $g \in \{b, m\}$, as

$$V_i^b(d|\omega) = \begin{cases} v_i \text{ if } d = \delta_i \\ 0 \text{ else,} \end{cases}$$

where $g = b$ indicates that group i is biased and so V_i^b gives the payoffs to a biased group for each decision, note that it is independent of ω. Correspondingly, for a moderate group let $g = m$ and define

$$V_i^m(d|\omega) = \begin{cases} v_i \text{ if } d = d_\omega, \\ 0 \text{ else.} \end{cases}$$

Define the expected utility for filing f_j to group i after observing signal s_i as

$$U_i(f_j|s_i) = \sum_{o^{-i} \in \mathcal{O}_{-i}} \Bigg[\mu_i(s_i) \Big[V_i^g(d_1|1) P_o^\sigma(f_j) + V_i^g(d_0|1)(1 - P_o^\sigma(f_j)) \Big] + \\ (1 - \mu_i(s_i)) \Big[V_i^g(d_1|0) P_o^\sigma(f_j) + V_i^g(d_0|0)(1 - P_o^\sigma(f_j)) \Big] \Bigg] P\Big(o^{-i}|s_i\Big) - c_i.$$

Define the expected utility for not filing to group i with preference δ_i after observing signal s_i as

$$U_i(n|s_i) = \sum_{o^{-i} \in \mathcal{O}_{-i}} \Bigg[\mu(s_i) \Big[V_i^g(d_1|1) P\Big(d = d_1|\mu(n, o^{-i})\Big) + V_i^g(d_0|1)\Big(1 - P\Big(d = d_1|\mu(n, o^{-i})\Big)\Big) \Big] + \\ (1 - \mu(s_i)) \Big[V_i^g(d_1|0) P\Big(d = d_1|\mu(n, o^{-i})\Big) + V_i^g(d_0|0)\Big(1 - P\Big(d = d_1|\mu(n, o^{-i})\Big)\Big) \Big] \Bigg] P\Big(o^{-i}|s_i\Big).$$

To show that an equilibrium in cut-point strategies exists, define the vector-valued mapping

$$\psi = (\psi_1(c; 0), \psi_1(c; 1), ..., \psi_N(c; 0), \psi_N(c; 1)) : [0, C]^{2N} \to [0, C]^{2N}.$$

Specifically, for group i we define $\psi_i(c; s_i)$ as

$$\psi_i(c; s_i) = \max\{0, U_i(f|s_i) - U_i(n|s_i) + c_i\},$$

for all $i \in N$. If group i is biased in favor of decision d_j then $f = f_j$. On the other hand, if group i is moderate then $f = f_{s_i}$.

As each ψ_i is summing over components that are continuous in $\hat{c}$ we know that each component of the vector-valued mapping ψ is continuous in $\hat{c}$, and thus ψ is continuous in c. Since $[0, C]^{2N}$ is compact and convex Brouwer's theorem yields a fixed point $\bar{c} = \psi(\bar{c})$. By definition, each $\psi_i(\omega)$ is player i's indifference condition between filing and not filing. As player i's utility for filing is strictly decreasing in c_i, no actor will want to deviate from filing when $c_i \leq \bar{c}_i(\omega)$ and not filing when $c_i > \bar{c}_i(\omega)$. Thus, $\bar{c}$ is an equilibrium.

We now show that groups do not want to deviate by filing in favor of a different decision. First, consider a group biased in favor of decision d_1. If it deviates and files in favor of d_0 this is off-the-path of play. Thus, assigning any belief such that $Pr(s_i = 1|f_0) < Pr(s_i = 1|f_1)$ is sufficient to prevent the group from deviating. Next, consider a moderate group that has observed $s_i = 1$. As it has observed $s_i = 1$ and $\pi_i > q$ its expected utility for d_1 is greater than its expected utility for d_0. As the judge's expectation is that $s_i = 1$ if $a_i = f_1$ and is that $s_i = 0$ if $a_i = f_0$ we have, for any realization of outcomes for the other groups, o^{-i}, that $\mu_J(f_1, o^{-i}) > \mu_J(f_0, o^{-i})$. Therefore, taking expectations over outcomes yields that the probability $d = d_1$ is strictly greater if i chooses f_1 over f_0 and so if group i is moderate and files it will not deviate from filing in favor of its signal.

What remains to be shown is that in a cut-point equilibrium for a biased group each pairing $(\bar{c}_i(0), \bar{c}_i(1))$ is ordered according to proposition 1. Consider a group i with preference $\delta_i = d_1$. In this case, we want that $\bar{c}_i(1) > \bar{c}_i(0)$. We prove this claim by contradiction. Assume that $\bar{c}_i(1) \leq \bar{c}_i(0)$. In this case, $\bar{\epsilon}_{\mu(f,o^{-i})} \geq \bar{\epsilon}_{\mu(n,o^{-i})}$. However, for $c_i \in (\bar{c}_i(1), \bar{c}_i(0))$ group i could switch from filing in state 0 to not filing, which would save on filing costs and increase the probability that its preferred decision is made. Thus, i has a profitable deviation which contradicts that $\bar{c}$ is an equilibrium.

Finally, we demonstrate that any perfect Bayesian equilibrium must be in cut-point strategies. Let $\sigma = (\sigma_1, ..., \sigma_N)$ be an arbitrary strategy profile with $\sigma_i : [0, \overline{C}] \times \{0, 1\} \rightarrow \Delta(\{d_0, d_1\})$. Thus, σ_i is a possibly mixed strategy for i. Let σ_{-i} be a strategy profile for all groups in N besides i. Assume group i is biased in favor of decision d_1. Given σ_{-i}, and assuming that the judge conjectures group i uses strategy $\hat{\sigma}_i$ after observing outcome $o \in \mathcal{O}$ the judge has belief $\mu(o)$ that $\omega = 1$, updated by Bayes' rule whenever possible. From our earlier analysis, it is clear that this yields some $\bar{\epsilon}(\mu(o))$ such that if $\epsilon > \bar{\epsilon}(\mu(o))$ then the judge chooses $d = d_1$ and if $\epsilon < \bar{\epsilon}(\mu(o))$ then the judge chooses $d = d_0$. Thus, given the distribution of the shock on $[-1, 1]$ according to F, this yields for each outcome o a corresponding probability that each decision is made denoted $Pr(d_1|o)$ and $Pr(d_0|o)$, which depend implicitly on σ_{-i} and $\hat{\sigma}_i$. Given strategy profile σ_{-i}, the prior belief on ω, the accuracy π_j of each group's signal, and the distribution of costs for each group G_j are fixed, let $Q(o|\sigma_{-i})$ denote the probability of outcome o. Then group i's expected utility for filing is

$$\sum_{o_{-i}\in\mathcal{O}_{-i}} Pr(d_1|o_{-i}, f)Q(o_{-i}|\sigma_{-i}) - c_i,$$

while its expected utility for not filing is

$$\sum_{o_{-i}\in\mathcal{O}_{-i}} Pr(d_1|o_{-i}, n)Q(o_{-i}|\sigma_{-i}).$$

Thus, it is a best-response for group i to file if and only if

$$\sum_{o_{-i}\in\mathcal{O}_{-i}} Pr(d_1|o_{-i}, f)Q(o_{-i}|\sigma_{-i}) - \sum_{o_{-i}\in\mathcal{O}_{-i}} Pr(d_1|o_{-i}, n)Q(o_{-i}|\sigma_{-i}) \geq c_i.$$

Similar arguments show that if the group is moderate or biased in favor of d_0 then their decision to file, given any strategies for the other group, has a similar characterization.

Thus, each group's best response to any strategy profile (and conjectured strategies by the judge) is in cut-points. As equilibrium strategies are best responses, it follows that in any equilibrium strategies must have this cut-point form.

Example: Free-riding groups

We now consider a simple example of our model to highlight that groups may have an incentive to free-ride on the filings of other groups with the same bias. Let $N = 2$, $\delta_1 = \delta_2 = d_1$, $\pi_1 = \pi_2 = 1$, $v_1 = v_2 = 1$, $p_1 = p_2 = 1$, $C = 1$, $\beta = 0$, $q = .5$ and let G be the uniform distribution.

As $p_i = 1$ neither group will ever file in a cut-point equilibrium if $\omega = 0$. Thus, $c_1(0) = c_2(0) = 0$. Using this and our analysis of judicial behavior we can write out the indifference conditions to solve for the equilibrium cut-points when $\omega = 1$ for each group as

$$c_1(1) = 1 - \hat{c}_2(1) - (1 - \hat{c}_2(1))\mu_J(n, n),$$
$$c_2(1) = 1 - \hat{c}_1(1) - (1 - \hat{c}_1(1))\mu_J(n, n).$$

Given the groups are using cut-point strategies the judge's belief after observing neither group file is

$$\mu_J(n, n) = \frac{(1 - \hat{c}_1(1))(1 - \hat{c}_2(1))}{1 + (1 - \hat{c}_1(1))(1 - \hat{c}_2(1))}.$$

Substituting this into the equilibrium conditions and solving the system of equations yields three cut-point point equilibrium. The first is a symmetric equilibrium in which $c_1(1) = c_2(1) = .43$. In the second, group 1 takes the lead and always files while group 2 free-rides and never files, $c_1(1) = 1$ and $c_2(1) = 0$. Finally, the reverse holds in the third equilibrium, with group 1 free-riding on group 2, $c_1(1) = 0$ and $c_2(1) = 1$.

Example: Competing groups

We now analyze cut-point equilibrium with two groups when the groups have opposite bias. Let $N = 2$, $\delta_1 = d_1$, $\delta_2 = d_0$, $\pi_1 = \pi_2 = 1$, $v_1 = v_2 = .6$, $p_1 = p_2 = .5$, $C = 1$, $\beta = 0$, $q = .5$ and let G be the uniform distribution. In this case, in a symmetric equilibrium the judge's belief following (f, f) and (n, n) is $\mu_J = .5$. The judge's belief following (f, n) and (n, f) are given by

$$\mu_J(f,n) = \frac{\overline{c}_1(1)(1-\overline{c}_1(1))}{\overline{c}_1(1)(1-\overline{c}_1(1)) + \overline{c}_1(0)(1-\overline{c}_2(0))} \text{ and}$$
$$\mu_J(n,f) = \frac{(1-\overline{c}_1(1))\overline{c}_1(1)}{(1-\overline{c}_1(1))\overline{c}_1(1) + (1-\overline{c}_1(0))\overline{c}_2(0)}.$$

Using this and substituting into the equilibrium filing equations we get $\overline{c}_1(1) = \overline{c}_2(0) \approx .43$ and $\overline{c}_1(0) = \overline{c}_2(1) \approx .03$. Taking these cut-points, it is straight-forward to calculate the probability that the judge makes the correct decision as $\approx .7$.

Example: Judicial welfare and group bias

Consider the parameters from the previous example. However, assume there is only one group. If that group is biased, then using the equilibrium equations and setting the parameters we get that a unique cut-point exists in which $\overline{c}_1(1) \approx .33$ and $\overline{c}_1(0) \approx .03$. These cut-points, along with the judge's updated beliefs after observing the group file and not file, we get that the probability the judge makes the correct decision with one biased group is $\approx .58$. Now assume that instead the group is moderate. In this case, substituting into the group's equilibrium equations yields $\overline{c}_1(1) = \overline{c}_1(0) = .1$. Calculating the probability the judge makes the correct decision in this case yields $.55 < .58$. Thus, the judge is more likely to decide correctly when the group is biased. Note that this is for the case when $p = 1/2$; for higher p the effect is more pronounced as the biased group filing strategy becomes even more informative. However, for p sufficiently low the opposite conclusion holds, with there not existing an informative equilibrium when the group is biased for $p = 0$. Also, note that $.58 < .7$, indicating that under these parameters adding a second competing group is beneficial for the judge.

Now re-consider the parameters from the example with two groups that share a bias and consider how changing group bias and the number of groups changes the probability the judge makes the correct decision. With one biased group we get $\bar{c}_1(1) = 1$ and $\bar{c}_1(0) = 0$. Thus, the judge always decides correctly. On the other hand, if there is one moderate group we get $\bar{c}_1(1) = \bar{c}_1(0) = 1/2$, and the judge makes the correct choice with probability .75.

Turning to multiple groups and solving for cut-point equilibrium with two competing groups under these parameters yields $\bar{c}_1(1) = \bar{c}_2(0) = 1/2$ and $\bar{c}_1(0) = \bar{c}_2(1) = 0$. Under these strategies the judge makes the correct decision with probability .75. Finally, recall that with two groups that share a bias we have multiple cut-point equilibria. If the groups play the equilibrium in which one group always files and the other never files this is equivalent to the one bias group model and, hence, the judge will always make the correct decision. On the other hand, if the groups end up in the equilibrium where $\bar{c}_1(1) = \bar{c}_2(1) \approx .43$ and $\bar{c}_1(0) = \bar{c}_2(0) = 0$ then the judge only makes the correct decision with probability $\approx .72$.

Therefore, under these conditions only having one biased group is always at least weakly better for the judge than any other arrangement. Interestingly, having competing biased groups or one moderate group works out equally well, although it is inferior to one biased group. This is because the effect of competition on filing behavior has a balancing effect on the judge's belief following no filings that is similar to how the judge updates if a moderate does not file. Finally, if there are two groups with similar ideologies how well the judge does depends on if the groups are able to coordinate on filings or not. If the groups do not coordinate on a "free-riding" equilibrium then the judge is strictly worse off under this arrangement compared to any other. On the other hand, if one group always takes a step back and the other group is always the filer then multiple friendly groups works as well as if there was only one biased group.

Group Bias and Influence

Proof of Lemma 1

Fix a cut-point equilibrium and consider a group i biased in favor of d_1. For any realization of outcomes for the other groups, o^{-i}, because $\bar{c}(1) > \bar{c}(0)$ if the judge observes that group i filed by Bayes' rule $\mu_J(f_1, o^{-i}) > \mu_J(n, o^{-i})$. Furthermore, by $\bar{c}(1) > \bar{c}(0)$ if the judge audits group i's brief then, having only observed f_1, in expectation the judge is more likely to learn that $s_i = 1$ rather than $s_i = 0$. Because $\bar{\epsilon}$ is strictly decreasing in μ_J this yields $Pr(d = d_1|f_1, o^{-i}) > Pr(d = d_1|n, o^{-i})$ for realization o^{-i}. As this holds for every outcome o^{-i} taking expectations over outcomes yields $Pr(d = d_1|f_1) > Pr(d = d_1|n)$ as required.

Similarly, consider a moderate group that observes $s_i = 1$. If we observe that the group files in favor of decision d_1 then the judge updates that group i's signal is $s_i = 1$. On the other hand, because $c_i(1) < C$ if the group does not file then the judge has some uncertainty over whether the group's signal is $s_i = 1$ or $s_i = 0$. Therefore, for any realization of outcomes o^{-i} we have $\mu_J(f_1, o^{-i}) > \mu_J(n, o^{-i})$. Because this holds for any realization o^{-i} taking expectations over the other group's outcomes we get that if group i observes $s_i = 1$ then the probability $d = d_1$ when the group files is greater than the probability $d = d_1$ if it does not.

Analogous proofs show the result if group i is biased for d_0 or is moderate and observes $s_i = 0$.

Proof Proposition 2

Recall that, in a cutpoint equilibrium, the judge's belief after observing a moderate group file in favor of d_1 is the same as when the judge observes the moderate group's signal directly. That is, for a moderate group i, the judge's belief after observing the moderate group file is equal to $Pr(\omega = 1|\tilde{o}_i = 1)$, as defined in the proof of proposition 1.

Next, recall that the judge's belief is equal to $Pr(\omega = 1|\tilde{o}_i = f)$, as defined in the proof of proposition 1, after a biased group files in favor of d_1. Finally, note that $\pi_i > \max\{q, 1-q\}$ and $\overline{c}_i(1) \geq \overline{c}_i(0)$ for a group biased in favor of d_1 implies $Pr(\omega = 1|\tilde{o}_i = 1) \geq Pr(\omega = 1|\tilde{o}_i = f)$ which yields the required result.

Proof of Proposition 3

We prove the proposition in three parts. First, we show that as $p_i \to 0$, equilibrium influence for biased groups also approaches 0. Second, we show that for p_i sufficiently high, biased groups do not file when they receive a signal contrary to their bias. Third, we suppose the group is moderate and show that if σ is a cut-point equilibrium given p_i then it is still a cut-point equilibrium for any $p'_i \neq p_i$.

For the first two steps, let group i be biased in favor of outcome δ_i. The difference in cut-points for group i is given by

$$\left|\overline{c}_i^*(1) - \overline{c}_i^*(0)\right| = \left|[U_i(f_{\delta_i}|s_1 = 1) - U_i(n|s_i = 1)] - [U_i(f_{\delta_i}|s_i = 0) - U_i(n|s_i = 0)]\right|. \quad (1)$$

To establish the first part of the proposition, we show that an equilibrium exists in which

$|\overline{c}_i^*(1) - \overline{c}_i^*(0)| = 0$. First, rewrite equation (1) as

$$\Big|[U_i(f_{\delta_i}|s_1 = 1) - U_i(f_{\delta_i}|s_i = 0)] - [U_i(n|s_i = 0) - U_i(n|s_i = 1)]\Big|. \tag{2}$$

Assume the judge and other groups conjecture that the cut-points used by group i are such that $|\hat{c}_i(1) - \hat{c}_i(0)| = 0$. In this case, the judge's belief over ω is the same, regardless if group i files or not. Furthermore, the other groups expect that group i's decision has no influence over outcomes. Therefore, the actions of the other players and thus outcomes, are independent of group i's decision. Consequently, group i's utility for filing is the same regardless of its signal and so the left term in brackets in equation (2) is 0. Similarly, the utility to the group for not filing is the same following either signal. As such, equation (2) is equal to 0 and so, consistent with the judge's and other groups' conjecture, group i's decision to file or not is independent of its signal. Hence, it is an equilibrium. Furthermore, in such an equilibrium because group i does not influence outcomes it can never be optimal to pay any positive cost c_i to file. Thus, it must be that $\overline{c}_i(1) = \overline{c}_i(0) = 0$. Because at $p_i = 0$ there exists such an equilibrium, upper hemicontinuity ensures that as $p_i \to 0$, there exist a sequence of equilibria in which $|\overline{c}_1(0) - \overline{c}_1(1)| \to 0$. Furthermore, as $p_i \to 0$, both $\overline{c}_1(1) \to 0$ and $\overline{c}_1(0) \to 0$. This establishes the first part of the proposition.

Second, we continue to assume that group i is biased in favor of outcome δ_i. Further, let $p_i = 1$, and suppose that group i has received signal $s_i \neq \delta_i$. In this case,

$$\overline{c}_i(0) = \sum_{o^k \in \mathcal{O}_{-i}} v_i \left[P\Big(d = \delta_i|\mu(s_i, o^k)\Big) - P\Big(d = \delta_i|\mu(n, o^k)\Big) \right] P\Big(o^k|s_i = 0\Big).$$

Following any action by the group the judge's belief cannot shift by more than it would had she actually observed the group's signal. Thus, $P\Big(d = \delta_i|\mu(s_i, o^k)\Big) - P\Big(d = \delta_i|\mu(n, o^k)\Big) < 0$. Because the group's gain following any outcome is always strictly negative it must be that at $p_i = 1$ we have $\overline{c}_i(0) = 0$. Further, because the inequality is strict, this implies that there exists some $\overline{p}_i < 1$ such that for all $p_i > \overline{p}_i$, $\overline{c}_i(0) = 0$. This establishes the second part of the result.

For the third and final part of the proposition, assume that group i is moderate. Recall that in a cut-point filing equilibrium, moderate groups always include factual information in their briefs and file consistent with the signal they receive. These two features imply that the judge's belief after observing a moderate group file in favor of d_i, $\mu(d_i, o)$ is equal to the judge's belief after observing the moderate group's signal directly, $\mu(s_i, o)$. This implies that, in a strategy profile σ, the judge's decision is constant in p_i for a moderate group.

To complete the proof, consider a vector of extraction probabilities $p = (p^M, p^B)$, where

p^M are the probabilities assigned to the moderate groups and p^B are the probabilities assigned to the biased groups. Denote another vector of extraction probabilities $p' = (p'^M, p^B)$, which only differs in the extraction probabilities of the moderate groups. Additionally, let $U_i(a|\sigma, p)$ be player i's utility for action a in assessment σ given extraction probabilities p. Finally, the fact that the judge's decision is constant in p_i for moderate groups implies that that for every p and p',

$$U_i(a_i|\sigma, p) = U_i(a_i|\sigma, p')$$

for all players i and actions a_i. Therefore, if σ is an equilibrium assessment under p, it is an equilibrium assessment under p'.

Proof of Proposition 4

Recall that in equilibrium

$$\sum_{o^k \in \mathcal{O}_{-i}} v_i \left[p_i P\Big(d = \delta_i | \mu(s_i, o^k)\Big) + (1 - p_i) P\Big(d = \delta_i | \mu(f, o^k)\Big) - P\Big(d = \delta_i | \mu(n, o^k)\Big) \right] P\Big(o^k | s_i\Big) = \bar{c}_i(s_i). \tag{3}$$

Now, suppose that $\pi_i = 1/2$. Note that when $\pi_i = 1/2$, s_i carries no information about w. This implies that for every pair of outcomes for player i o_i, o_i', and every profile of outcomes for the other players o_{-i},

$$\mu(o_i, o_{-i}) = \mu(o_i', o_{-i}).$$

This implies that for every pair of outcomes for player i o_i, o_i', and for every profile of outcomes for the other players o_{-i},

$$P\Big(d = \delta_i | \mu(o_i, o_{-i})\Big) - P\Big(d = \delta_i | \mu(o_i', o_{-i})\Big).$$

From this, it follows that the left hand side of equation 3 is equal to 0, which establishes the result.

Empirically Assessing Influence

In this section, we provide formal proof of our results on empirically assessing the influence of amicus filings.

Proof of Lemma 2

That the group has strictly positive influence if $v > 0$ follows from lemma 1.

The second part of the result follows from application of Bayes rule and the fact that in a cut-point filng equilibrium, a group never files if $v_i = 0$.

Proof of Proposition 5

Before proving the result, it is useful to establish some notation. Suppose that we possess data on all cases heard, as well as whether the group filed in each particular case. Let the group's average influence level across a dataset of M cases be defined as :

$$\frac{1}{M}\sum_{k=1}^{M}\Big[Pr(d = d_1| \text{ group files in case } k) - Pr(d = d_1| \text{ group does not file in case } k)\Big]$$

As, from lemma 2, group influence varies based on whether the group values a ruling in its favor or not, we can decompose influence among cases relevant and not relevant for the group. Let v^k be the group's value for case k and define $R \subseteq M$ as the set of relevant cases ($v^k > 0$) and $I \subseteq N$ as the set of irrelevant cases ($v^k = 0$). We can now rewrite the group's average influence as:

$$\frac{1}{M}\Big(\sum_{k\in R}\Big[Pr(d = d_1|f^k, k \in R) - Pr(d = d_1|n^k, k \in R)\Big] + \sum_{k\in I}\Big[Pr(d = d_1|f^k, k \in I) - Pr(d = d_1|n^k, k \in I)\Big]\Big)$$

where, for notational simplicity, $Pr(d = \delta|f^k)$ and $Pr(d = \delta|n^k)$ denote the probabilities the judge issues the group's preferred ruling given that it does and does not file on case k, respectively. By lemma 2, the second summation is equal to 0, as influence on cases that are not relevant is equal to 0. We can therefore rewrite the average level of a group's influence as

$$\frac{1}{M}\sum_{k\in R}\Big[Pr(d = d_1|f^k, k \in R) - Pr(d = d_1|n^k, k \in R)\Big].$$

With this notation in hand, we can move along to proving the proposition. Recall that groups never file when a case is not relevant. This implies that in expectation, the first term in $\hat{I}$ is a consistent estimate of $Pr(d = d_1|f_i, i \in R)$. To calculate influence we also wish to estimate $Pr(d = d_1|n, i \in R)$. However, if both relevant and irrelevant cases are pooled, taking the mean outcome when a group does not file results in an estimate that is in expectation equal to $Pr(i \in R)Pr(d = d_1|n, i \in R) + Pr(i \in I)Pr(d = d_1|n, i \in I)$. Note that, because the judge does not update after observing a non-filing when a case is irrelevant to a group's interests, $Pr(d = d_1|n, i \in R) < Pr(d = d_1|n, i \in I)$, which establishes the result.

Endogenous Auditing

Here, we analyze how the influence of briefs is altered if the judge strategically chooses whether or not to read a filing. Our baseline model abstracted from such auditing to incorporate other aspects of the amicus environment important to understanding the trade-offs faced by groups when filing and in a manner tractable enough to analyze the effectiveness of briefs.[23] To build in endogenous auditing, we focus on the case with one group here and study examples with two groups in the appendix.

In doing so, we find that our conclusions about moderate groups from the exogenous auditing continue to hold, and that allowing for endogenous auditing provides a deeper understanding of these insights. Specifically, the endogenous auditing of briefs by the courts differentially effects influence depending on whether the group is biased or moderate. Similar to our baseline results, moderate group influence is not sensitive to endogenous auditing, while biased group influence grows when the judge is able to strategically read briefs. Additionally, which briefs the judge wants to allocate her time and resources toward depends on filing group characteristics.

Consider an altered version of our amicus model with one group in which judges choose whether to read a brief or not. Rather than assuming that the judge reads group i's brief with probability p_i, we now model the judge's decision to read a brief as an endogenous choice. Formally, we assume that, after a group chooses whether to file, the judge learns her private cost c_J, which is drawn from distribution $G_J(c_J)$. After learning this cost, the judge chooses to either read the brief, paying cost c_J and learning the brief's factual content, or does not read the brief, paying no cost and remaining ignorant of the filing's factual content. After this, the game continues per the model previously studied, with the judge receiving a shock to her utility and issuing a ruling.

As foreshadowed, the impact of auditing is qualitatively different depending on whether the group in question is moderate or biased. The next result establishes this differential effect that endogenous auditing of filings has on group influence depending on group ideological orientation.[24]

[23] An additional interpretation of the baseline model is that group reputations are relatively fixed at the time of the case and, thus, the judge commits to reading briefs from some groups with a higher probability, i.e., she directs her clerks ahead of time to focus on briefs from certain groups should they file.

[24] In studying endogenous auditing we focus on equilibrium that can be characterized using cut-point strategies similar to those in the baseline analysis. Importantly, cut-point equilibria continue exist in the

Proposition 6. ***(Group influence and auditing)***

1. *If the group is biased then its influence is positive when the judge has the ability to audit. However, if the judge could never audit then it has no influence.*

2. *If the group is moderate then its influence is the same under both strategic auditing and no auditing.*

This result indicates that allowing the judge to choose which briefs to read carefully, rather than assuming an exogenous probability of reading, can encourage more informative and more influential filing behavior. However, this effect is not uniform across groups. Only biased group behavior changes when the judge chooses whether or not to read a brief. Interestingly, analogous to previous results, a change in influence comes through direct and indirect channels. Information is translated directly when a judge reads a group's brief, and is transferred indirectly through the group's strategic filing decisions. Our analysis shows that the direct channel is necessary for the indirect channel to operate for biased groups, as such groups will only have influence if there is some threat that the judge will read their briefs. In contrast, this is not true for moderates, as their filings are informative even when there is no expectation that the judge will verify filing content.

The endogenous auditing model also provides insights into which briefs the judge will spend time and effort reading closely. The judge targets different briefs depending on filing group characteristics. Also, when we analyze special cases of our model with two groups to further understand judicial incentives we find that, consistent with proposition 6, the judge may spend time reading a brief submitted by a biased group but never a moderate group (see appendix). The moderate group reveals its signal through which side its brief supports, so the judge need not expend resources assessing the details of the group's filing. Intuitively, we also find that if two biased groups file a brief, but one group's signal is much more accurate, then the judge targets the high quality group.

First, we establish existence of a cut-point equilibrium in the model with endogenous auditing. We use an argument similar to the one used to establish equilibrium existence in the baseline model, constructing a mapping that represents the group's and the judge's indifference conditions for filing and auditing, respectively. Verifying continuity of this mapping delivers existence via Brouwer's fixed point theorem.

Assume that the group uses a cut-point strategy when deciding whether to file or not. Given this cutpoint strategy, we define notation for the judge's beliefs, obtained via Bayes

cases we analyze with endogenous auditing (see appendix).

rule, so that μ_o denotes the Judge's belief after outcome o. More specifically, let

$$\mu_1 = \frac{\pi_i q}{\pi_i q + (1-\pi_i)(1-q)}$$

denote the judge's belief if she audits and learns that the group's signal is equal to 1. Similarly, let

$$\mu_0 = \frac{\pi_i(1-q)}{\pi_i(1-q) + (1-\pi_i)q}$$

be the judge's belief if she audits and learns that the group's signal is equal to 0. If the judge does not audit and the group files playing a cut-point strategy, then the judge's belief is equal to

$$\mu_f = \frac{q\Big(\pi_i G(\overline{c}_i(1)) + (1-\pi_i)G(\overline{c}_i(0))\Big)}{q\Big(\pi_i G(\overline{c}_i(1)) + (1-\pi_i)G(\overline{c}_i(0)\Big) + (1-q)\Big(\pi_i G(\overline{c}_i(0)) + (1-\pi_i)G(\overline{c}_i(1)\Big)}.$$

Finally, if the group does not file when the group uses a cut-point strategy, the judge's belief is equal to

$$\mu_n = \frac{q\Big(\pi_i G(\overline{c}_i(1)) + (1-\pi_i)G(\overline{c}_i(0))\Big)}{q\Big(\pi_i(1-G(\overline{c}_i(1))) + (1-\pi_i)(1-G(\overline{c}_i(0)))\Big) + (1-q)\Big(\pi_i(1-G(\overline{c}_i(0))) + (1-\pi_i)(1-G(\overline{c}_i(1)))\Big)}.$$

Next, recall that, given belief μ_o, the judge will choose decision d_1 if the shock is above cutpoint $\overline{\epsilon}_o$, which itself is a function of μ_0 as described earlier. The judge's expected utility for not auditing, given that the group files is given by

$$U_J(N|f_j) = \Big(\frac{\overline{\epsilon}_{f_j}+1}{2}\Big)\Big(\mu_{f_j} + \beta + \frac{\overline{\epsilon}_{f_j}+1}{2}\Big) + \Big(\frac{1-\overline{\epsilon}_{f_j}}{2}\Big)\Big(1-\mu_{f_j}\Big).$$

The judge's expected utility for auditing is

$$\begin{aligned} U_J(A|f_j) = & \\ & \mu_{f_j}\Bigg[\Big(\frac{\overline{\epsilon}_1+1}{2}\Big)\Big(\mu_1 + \beta + \frac{\overline{\epsilon}_1+1}{2}\Big) + \Big(\frac{1-\overline{\epsilon}_1}{2}\Big)\Big(1-\mu_1\Big)\Bigg] + \\ & (1-\mu_{f_j})\Bigg[\Big(\frac{\overline{\epsilon}_0+1}{2}\Big)\Big(\mu_0 + \beta + \frac{\overline{\epsilon}_0+1}{2}\Big) + \Big(\frac{1-\overline{\epsilon}_0}{2}\Big)\Big(1-\mu_0\Big)\Bigg] - c_J. \end{aligned}$$

Comparing expected utilities yields that after seeing the group file in favor of decision j the judge audits if her cost is below some cut-point $\overline{c}(j)$ and does not audit if her cost is above this cut-point. Note that because the group's costs are continuously distributed, the judge's utility for auditing and not auditing are each continuous in the group's cut-points $\bar{c}(0)$ and $\bar{c}(1)$.

Next, consider the group's utility. Using a notation similar to before, let $P^{\sigma}(f_j)$ be the probability that the judge implements decision d_1, given that the judge's auditing cutpoints are $\bar{c}_J(0)$ and $\bar{c}_J(1)$ when the group offers filing f_j and has received signal s_i. This probability is

$$P_J^{\sigma}(f_j) = \left[G_J(\bar{c}_J(f_j))P\Big(d = d_1|\mu_{s_i}\Big) + \Big(1 - G_J(\bar{c}_J(f_j))\Big)P\Big(d = d_1|\mu_{f_j}\Big)\right].$$

With this, we substitute $P_J^{\sigma}(f_j)$, into the group indifference conditions from the proof of proposition 1, replacing $P_0^{\sigma}(f_j)$. Because $P_J^{\sigma}(f_j)$ is continuous in the vector of auditing and filing cutpoints $\bar{c}$, the same argument from Proposition 1 delivers a fixed point via Brouwer's theorem, ensuring existence of a cut-point equilibrium.

Proof of Proposition 6

The first statement in part 1 follows from the characterization of cut-point equilibrium with endogenous auditing. To establish the second statement, note that when the judge is unable to audit, the model is equivalent to the baseline model where $p_i = 0$. Proposition 3 implies that influence is 0 in this case, which establishes the result.

Now, we prove part 2. We show that a group's influence is the same under both strategic auditing and no auditing. To establish the result, we demonstrate that the judge never audits a moderate group in equilibrium.

To deduce a contradiction, suppose not. That is, suppose that the judge faces a filing from a moderate group, and the judge audits for some positive cost $c_J > 0$. Note that by definition of the moderate group's strategy, the judge knows the group's signal with certainty, conditional upon observing them file. This implies that $U_J(A) < U_J(N)$, contradicting the assumption that the judge audits for some $c_J > 0$. Therefore, the judge never audits a moderate group with positive probability, which implies the result.

Example: Targeting a high quality group

Consider $N = 2$ with both groups being biased and $\pi_1 = 1 > \pi_2$. If only group 1 files then re-using our earlier analyses and replacing $\mu_J(f_j)$ with $\mu_J(f_j, n)$ yields an $\bar{c}_J(f_j, n)$ that is continuous in the groups' cut-points such that the judge audits group 1 if her costs is.

Similarly we obtain $\overline{c}_J(n, f_j)$ for group 2. Thus, we obtain

$$P_J^{\sigma}(f_j, n) = \left[G_J(\overline{c}_J(f_j, n)) P\Big(d = d_1 | \mu_J(s_i, n)\Big) + \Big(1 - G_J(\overline{c}_J(f_j, n))\Big) P\Big(d = d_1 | \mu(f_j, n)\Big) \right],$$

$$P_J^{\sigma}(n, f_j) = \left[G_J(\overline{c}_J(n, f_j)) P\Big(d = d_1 | \mu(n, s_i)\Big) + \Big(1 - G_J(\overline{c}_J(n, f_j))\Big) P\Big(d = d_1 | \mu(n, f_j)\Big) \right].$$

Moving on from the outcomes where only one group files, now consider if both file. Clearly if the judge is going to audit there is no reason to ever audit the lower quality group over the high quality group, as auditing the high quality group reveals the correct state for sure. Thus, in a cut-point equilibrium the judge will target the high quality group if both groups audit and to complete our analysis we just need to show that a cut-point equilibrium indeed exists. Therefore, if both groups file we only need to find when the judge prefers to audit group 1 over not auditing at all. In this case, her expected utility for auditing is given by

$$U_J(N | f_{\delta_1}, f_{\delta_2}) = \Big(\frac{\overline{\epsilon}(f_{\delta_1}, f_{\delta_2}) + 1}{2}\Big)\Big(\mu_J(f_{\delta_1}, f_{\delta_2}) + \beta + \frac{\overline{\epsilon}(f_{\delta_1}, f_{\delta_2}) + 1}{2}\Big) + \Big(\frac{1 - \overline{\epsilon}(f_{\delta_1}, f_{\delta_2})}{2}\Big)\Big(1 - \mu_J(f_{\delta_1}, f_{\delta_2})\Big).$$

On the other hand, the judge's expected utility for auditing group 1 is

$$\begin{aligned} U_J(A | f_{\delta_1}, f_{\delta_2}) = & \mu_J(f_{\delta_1}, f_{\delta_2}) \left[\Big(\frac{\overline{\epsilon}(1) + 1}{2}\Big)\Big(1 + \beta + \frac{\overline{\epsilon}(1) + 1}{2}\Big) \right] + \\ & (1 - \mu_J(f_{\delta_1}, f_{\delta_2})) \left[\Big(\frac{\overline{\epsilon}(0) + 1}{2}\Big)\Big(\beta + \frac{\overline{\epsilon}(0) + 1}{2}\Big) + \Big(\frac{1 - \overline{\epsilon}(0)}{2}\Big) \right] - c_J. \end{aligned}$$

Thus, we get that there is an $\overline{c}_J(f, f)$ such that if $c_J \leq \overline{c}_J(f, f)$ then the judge audits group 1, otherwise she does not audit either group. Furthermore, as the judge's belief is continuous in the cut-points $\hat{c}_i(1)$ and $\hat{c}_i(0)$ for $i \in \{1, 2\}$ we get that $\overline{c}_J$ is continuous in the groups' cut-points. Using this, we get

$$P_J^{\sigma}(f_{\delta_1}, f_{\delta_2}) = \left[G_J(\overline{c}_J(f_{\delta_1}, f_{\delta_2})) P\Big(d = d_1 | \mu_J = 1\Big) + \Big(1 - G_J(\overline{c}_J(f_{\delta_1}, f_{\delta_2}))\Big) P\Big(d = d_1 | \mu(f_{\delta_1}, f_{\delta_2})\Big) \right]$$

Substituting the new P_J^{σ}s into our original equilibrium equations yields existence of cut-point equilibrium.

Example: Targeting a biased group

Now consider the case where $N = 2$ and assume group 1 is in favor of d_1 and group 2 is moderate. Given a cut-point equilibrium, if the judge observes group 2 file in favor of decision d_j then the judge updates that $s_2 = j$. Thus, it is not optimal for the judge to pay a cost to audit group 2. If group 1 files then the judge's decision to audit following (f, n) and (f, f) can be described using equations similar to the previous case, which yields

$$P_J^{\sigma}(f_j, n) = \left[G_J(\bar{c}_J(f_j, n))P\Big(d = d_1|\mu_J(s_i, n)\Big) + \Big(1 - G_J(\bar{c}_J(f_j, n))\Big)P\Big(d = d_1|\mu(f_j, n)\Big) \right],$$

$$P_J^{\sigma}(f_{\delta_1}, f_{\delta_2}) = \left[G_J(\bar{c}_J(f_{\delta_1}, f_{\delta_2}))P\Big(d = d_1|\mu_J(s_1, f_j)\Big) + \Big(1 - G_J(\bar{c}_J(f_{\delta_1}, f_{\delta_2}))\Big)P\Big(d = d_1|\mu(f_{\delta_1}, f_{\delta_2})\Big) \right].$$

Substituting these in for group 1 in the original existence argument and setting $p_2 = 0$ for group 2 delivers existence of cut-point equilibrium in this model.

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