

Campaign Spending, Access, and Lobbying*

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Abstract

Interest groups can influence governmental policy through multiple channels. First, they may spend money before elections to help elect their preferred candidate. Second, they may also lobby after the election to affect the implemented policy. Politicians may grant access to politicians in exchange for electorally helpful contributions. We analyze a game-theoretic model elections which contain campaign spending and lobbying to understand the strategic relationship between these two means of outside influence. We find that both interest group and politician ideology predicts campaign spending. Spending patterns are different not only when an interest groups is more extreme or moderate than its preferred candidate, but also if the other interest group is more extreme or moderate than the opposing candidate. We find that extreme interest groups spend more than moderate ones on the same race. We also find that spending is inversely correlated with the probability of winning the election except when both interest groups are moderate.

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1 Introduction

The Koch brothers' PAC Americans for Prosperity pledged to spend 400 million dollars on the 2018 U.S. midterm elections (Easley, 2018). The campaign was designed to help their preferred candidates (mostly Republicans) win the elections. At the same, both brothers sit on the board of the American Legislative Executive Council (ALEC). ALEC writes model legislation for legislators to introduce, essentially a subsidy for adopting the preferred policy positions of the Kochs.

Seeking to affect policy through multiple channels is not unique to this one family. Tripathi et al. (2002) and Lake (2015) show that the most influential interest groups routinely engage in both campaign spending and on lobbying. Further, much of the ire towards *Citizens United v. FEC* tacitly presumes, correctly, that interest groups are engaged in both campaign spending and lobbying (Kirkpatrick, 2010).¹

Politicians understand this dynamic as well. Consider this quote from former congressman Mick Mulvaney: “We had a hierarchy in my office in Congress. If you’re a lobbyist who never gave us money, I didn’t talk to you. If you’re a lobbyist who gave us money, I might talk to you” (Stewart, 2018). Mulvaney is insinuating that his office would give preference to interest groups that had contributed to the campaign.

Campaign spending and lobbying affect policy in different but interrelated ways. Interest groups use campaign spending to help elect ideologically aligned politicians. After the election, interest groups may lobby the winning candidate, increasing the affinity of aligned politicians and (potentially) moderating the policy of less aligned politicians.

However, A plausible reading of the Mulvaney remarks is that contributions are a necessary but not sufficient condition for access to a politician, with ideological alignment being a likely additional necessary condition. In this setting, only the interest group aligned with the winning candidate may participate (along with the candidate herself) in lobbying. McKay

¹The article also discusses the role of interest group extremism as increasingly important given the enhanced role interest groups would certainly be taking.

(2018b) and Kalla and Broockman (2016) present more systematic evidence that campaign contributions buy access, showing that politicians do not just implement policies favored by supported interest groups but also use language specifically written by groups that hosted fundraisers. Further, Fourniaies and Hall (2018), Powell and Grimmer (2016), and McKay (2018a) show that interest group prioritize contributions for candidates that will hold positions on relevant committees.

Politicians control the access they give to interest groups after the election. They can use their discretion to influence how the groups spend money during the campaign. Therefore the amount of campaign spending is affected by behavior both before the election through the access design channel and after the election through the lobbying channel. We want to account for both channels in our analysis.

We seek to understand the strategic relationship between campaign spending and lobbying, two alternative means by which to influence policy outcomes. We employ a three-stage model of two interest groups, two candidates, and one representative voter. In the first stage, politicians commit to how much lobbying access they will give the aligned interest group if they win the election. The second stage is the electoral campaign. Interest groups may spend on the campaign for their aligned politician, after which a median voter chooses her preferred candidate. In the third stage, the voter decides which candidate gets to implement policy.

We find that both interest group and politician ideology predicts campaign spending. In particular, the relative *alignment* of both types of actors is of crucial importance. Spending patterns are different not only when an interest groups is more extreme or moderate than its preferred candidate, but also if the other interest group is more extreme or moderate than the opposing candidate.

For example, when there is no post-election lobbying, the opposing interest group's ideology does not impact the aligned interest groups campaign expenditures. However, once interest groups can lobby after the election, the extremity of opposing interest groups affects

how much the aligned interest group spends on the campaign. As the opposing interest group gets more extreme, they pull final policy to a more extreme location. This means the aligned interest is worse off if their candidate loses the election, and this pushes them to spend more money on the campaign today.

We also find that politicians only grant access to interest groups when their aligned interest group is extreme, but the opposing interest group is moderate. When both interest groups are extreme, their contributions cancel each other out. Therefore granting access confers no additional electoral benefits and is a waste. When an interest group is moderate, they do not contribute much more when granted access; this makes the tradeoff of better electoral chances in exchange for a policy loss not worth it. However, an aligned, extreme interest group is willing to pay more for access. When the opposing interest group is moderate, this increases the politician's chances of election and is worth giving up some policy.

We find that when one interest group is more moderate than their aligned candidate and the other group is more extreme, the extreme interest group always spends more than the moderate groups on the campaign. Losing the election is worse for the extreme group, and the correspondingly spend more on the campaign to try and stave off this outcome.

Next, we show that the extremity of interest groups and politicians have opposite effects on the election probability. As the aligned interest group becomes more extreme, election chances (weakly) increase as the interest groups spend more money on the campaign. However, as politicians become more extreme, their chance of winning the election decreases because policy moves farther away from the expected median voter. Extremity of the opposing politician and interest group work in the exact opposite directions as the aligned players.

Finally, we compare spending to the probability of winning the election and to the policy location. Again we find that relative alignment is crucial. For example, when both interest groups are extreme, spending is inversely correlated with policy moderation and the probability of election. However, when both interest groups are moderate, higher spending is

associated with policy moderation and a better chance of winning the election. This is true even though the final policy in both cases is exactly the same.

Our electoral framework uses a probabilistic voting model. The specific technology draws upon the uncertain median voter technology first used in Calvert (1985) and extended in Groseclose (2001), amongst others. Instead of focusing on candidate positioning, we focus instead on how interest groups respond to different candidate positions.

Our lobbying approach uses a reduced form of the equilibrium policies from Grossman and Helpman (2001) and Duggan and Gao (2019). In particular, we use a weighted average of the ideal points of the elected politician and their aligned interest group similar to the equilibrium contest functions used in these two papers.

Prior work has generally considered campaign spending or lobbying in isolation, rather than jointly. While Felli and Merlo (2007) allow interest groups to both support campaigns and lobby elected politicians, interest groups never use both tactics in equilibrium. Our paper shows how both campaign spending and lobbying on behalf of the same politician can occur, exploring conditions under which they respond similarly or differently to changes in exogenous factors. As stated above, the empirical literature shows that interest groups spend money on both campaigns and lobbying for the same politicians (Tripathi et al., 2002; Lake, 2015), which is consistent with an interpretation of our model.

One way to conceptualize our model is with the inside/outside lobbying dynamic of Wolton (2019). Campaign spending (perhaps best thought of as advertising in this model) influences the public as *outside* lobbying. The lobbying in the second stage of our model, however, directly influences the politician and would thus be an example of *inside* lobbying.

In contrast to our paper, some other models use the all-pay contest framework instead (or in addition to) the uncertain median voter framework of the current paper. Some of these papers include Meirowitz (2008) and Ashworth and Bueno de Mesquita (2009). However, in addition to the different modeling technology, these papers let politicians themselves make campaign investments. We depart by modeling campaign contributions from interest groups,

not politicians, as well as by adding a subsequent lobbying stage. Morton and Myerson (2012) do allow interest groups and politicians to contribute to campaigns, although they also do not analyze a second, lobbying stage. While we do not include politicians as strategic actors in the first stage, we do conceive of politicians as strategic actors in the second, lobbying stage.

Our model differs from previous models that focused on informational campaign contributions and lobbying (Cotton, 2012; Dahm and Porteiro, 2008; Schnakenberg and Turner, 2018; Bennedsen and Feldmann, 2006). We specifically focus on policy-location motivated interest groups, and not information. This allows us to be parsimonious in identifying the pure ideological effects of access and interest groups.

Other papers model both contributions and lobbying within a contributions-as-access framework (Cotton, 2009; Judd, 2019). Our approach focuses on how the politician can optimally set access given the known lobbying institution after the election. Another approach (seen in Judd (2019) for example) would be to let access efforts in the second stage depend upon contributions in the first stage.

This paper also contributes to the small but growing literature on interest group ideology and its policy implications. Bonica (2013, 2014) develops a procedure to measure interest group ideology, while McKay (2010) and Thieme (2019) show that more extreme interest groups spend more than moderate groups on campaigns. Brunell (2005) shows that while interest groups often give to both parties, their sincere giving is only targeted to one party. We propose that the empirical studies consider another factor: the ideological distance between interest groups and candidates.

The empirical literature has been mixed with regards to the effectiveness of political spending.² Ansolabehere et al. (2003), for example, advance the notion that contributions are not particularly effective, but may instead be a form of consumption. Our model helps

²While we focus mostly on the U.S. setting, the effects of political money are of interest world wide. Titl and Geys (2019), for example, show how donations to winning politicians increases a firm's chances of receiving procurement contracts in the Czech Republic.

rationalize the fact of relatively low amounts of observed money in politics (often known as Tullock’s Paradox), even when money is very effective, due to a canceling out effect under some lobbying regimes.

2 Model

Three distinct groups of players interact within the model: interest groups (G), politicians/candidates (P), and voters. For simplicity, we consider a single representative voter, often referred to as the median voter or simply the voter (M). All players have an ideal point on the real line. No platform commitment is possible by politicians.

The two candidates, one left-leaning candidate (P_L) and one right-leaning (P_R), have ideal points $\hat{x}_{P_L} \leq 0 \leq \hat{x}_{P_R}$. Similarly, there are two interest groups, one left-leaning (G_L) and one right-leaning (G_R), with ideal points $\hat{x}_{G_L} \leq 0 \leq \hat{x}_{G_R}$. We make no assumptions here about whether the politician ideal points are greater or less than the interest group ideal points. We call P_j and G_i *aligned* if $i = j$ and similarly we call P_j and G_i *opposed* if $i \neq j$.

The voter’s ideal point is drawn from a uniform distribution on $[-\frac{1}{2}, \frac{1}{2}]$. Before the election, the realization of the voter is unknown to politicians and interest groups. We assume that $\hat{x}_{P_L}, \hat{x}_{G_L} \leq -\frac{1}{2}$ and $\hat{x}_{P_R}, \hat{x}_{G_R} \geq \frac{1}{2}$. This makes the algebra easier, but is not essential for our results.

Policy is determined by a weighted average of the politician’s and aligned interest group’s ideal points. The final policy, denoted $x_{P_j}^*$, is determined by the following expression:

$$x_j^* = \frac{\alpha_j^G \hat{x}_{G_j} + \alpha_j^P \hat{x}_{P_j}}{\alpha_j^G + \alpha_j^P} \tag{1}$$

for $j \in \{L, R\}$

where α_j^G, α_j^P are weights that define how much influence aligned and misaligned interest groups have over policy when candidate j wins. We set $\alpha_j^P = 1$, so α_j^G determines how much relative influence the aligned interest group has if politician j wins the election.

The model has three stages: an access stage, a campaign stage, and an election stage. In the access, the politicians simultaneously decided how much access to grant their aligned interest group. In the campaign stage, the interest groups simultaneously make their campaign spending decisions, where $s_{G_i, P_j} \geq 0$ is the contribution of interest group G_i to P_j . Finally, the voter then chooses a candidate, and the final policy is realized. Figure 1 summarizes the sequences of play.

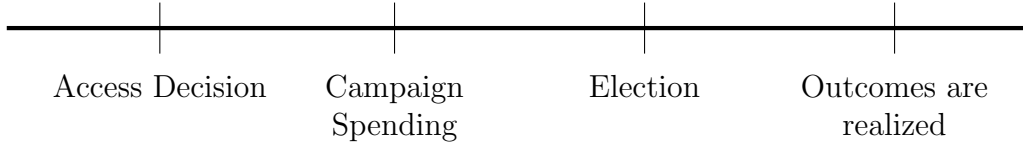


Figure 1: Sequence of Play

In addition to policy, the voter cares about politician valence. The total utility she receives if politician j wins the election with valence v_{P_j} and implements x_{P_j} is given by

$$U_M(x_{P_j}, v_{P_j}; \hat{x}_M) = -|x_{P_j} - \hat{x}_M| + v_{P_j} = -|x_{P_j}| + v_{P_j}.$$

Interest groups care about money spent on campaigns in addition to effort expended on lobbying and the policy that is ultimately implemented. Group i 's total utility from policy x_{P_j} being implemented and any campaign contributions made³ is given by:

$$U_{G_i}(x_{P_j}, \hat{x}_{G_i}) = -|x_{P_j} - \hat{x}_{G_i}| - \frac{v_{P_i}^2}{4}.$$

where $s_i = \frac{v_{P_i}^2}{4}$.⁴

Politicians only care about policy. Their utility is simply

$$U_{P_i}(x_{P_j}, \hat{x}_{P_i}) = -|x_{P_j} - \hat{x}_{P_i}|$$

³We assume that the interest group only contributes to the aligned politician

⁴We use this specific form for convenience. Any quadratic cost will give the same results.

2.1 Discussion and Assumptions

A few comments on our modeling assumptions. First, we assume that the politician receives no extra rents from being elected to office. This is to keep the focus purely on the policy motivations of all players. The addition of an office benefit makes giving access to the interest groups more attractive, but does not qualitatively change our results.

The opportunity cost of money spent during the campaign stage includes all manner of other activities in which interest groups engage; for example, they may fund raise, conduct membership drives, or increase salaries. We conceive of lobbying less as a monetary favor but rather as a legislative subsidy in the sense of Hall and Deardorff (2006). Interest groups can influence final policy with bill-writing, advice, expertise, and other non-monetary support.

We do not include a cost of lobbying for the special interest groups merely as a technical convenience. Adding a lobbying cost does not affect our qualitative results. In essence, we treat lobbying in reduced form way, purely through the change in final policy. We do not explicitly model the strategic behavior, but instead consider how different possible lobbying outcomes will affect the campaign stage.

We also assume that all ideal points are such that both politicians have a positive probability of winning the election. This is simply to ensure that there is interesting strategic behavior to analyze.

3 Analysis

3.1 Voter's Problem: Election

To solve this game, we proceed by backwards induction. The last mover is the voter, and therefore we first analyze how the voter chooses a politician. The voter elects the left politician over the right politician if the left politician's combination of policy and valence is

greater than the right politician's combination. Formally, the voter votes for politician L if

$$-|x_L^* - \hat{x}_M| + v_L > -|x_R^* - \hat{x}_M| + v_R$$

or

$$v_L > |x_L^* - \hat{x}_M| - |x_R^* - \hat{x}_M| + v_R \tag{2}$$

and randomly chooses one of the politician's if these expressions are equal.⁵

Note that the voter is concerned with the final policies, not with the politician or interest group ideal points per se. She understands the policy outcome is determined by the access granted to the interest group by the politician, and uses that knowledge when voting. As an example, consider a comparison between a world in which there is no post election lobbying and one where interest groups can lobby.

When there is no post-election lobbying, the winning politician implements her ideal point. Figures 2 and 3 show how policy is more moderate when politicians are closer to the expected median voter. The interest group ideal points have no effect on the final policy.

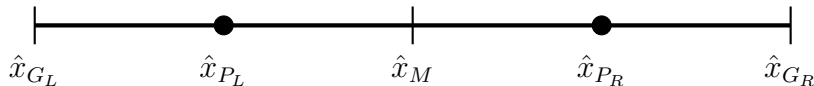


Figure 2: Location of policy after no lobbying for extreme interest groups

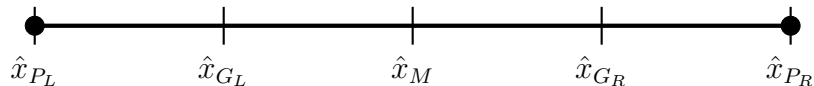


Figure 3: Location of policy after no lobbying for moderate interest groups

Lobbying can be beneficial or harmful to the voter. Figure 4 shows that extreme interest groups pull the final policy away from median voter, as there is no counterweight from the losing interest group.

⁵We will generally ignore the tie scenario as this happens with a probability of 0.

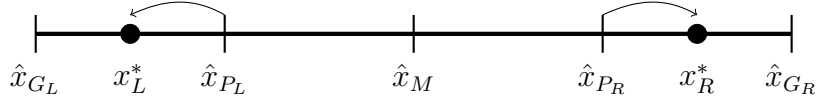


Figure 4: Location of policy after pure access lobbying for extreme interest groups

Moderate interest groups moderate an aligned but relatively extreme politician vis-à-vis the policy the politician would implement without any lobbying, as in Figure 5.

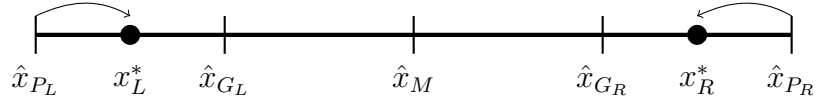


Figure 5: Location of policy after pure access lobbying for moderate interest groups

3.2 Interest Groups' Problem: Contributions

The Interest Groups' problem is more complicated. They do not know the identity of the voter; they only know the voter's ideal point in expectation. They need to choose the optimal valence level given the ideal points of the various players, the level of access they will receive after the election, and the level of access the other interest group would receive if the opposing politician wins the election.⁶

Why is one interest group concerned about the other interest group's access? Consider the two final possible policies: x_L^* and x_R^* . The expressions for these policies, $x_L^* = \frac{\alpha_L^G \hat{x}_{GL} + \hat{x}_{PL}}{\alpha_L^G + 1}$ and $x_R^* = \frac{\alpha_R^G \hat{x}_{GR} + \hat{x}_{PR}}{\alpha_R^G + 1}$ are functions of the interest group ideal points. The interest groups, like the voter, are concerned with final policies, and these final policies contain the interest group ideal points. Therefore the left interest group must care about the right group's ideal point and vice versa.

To optimally choose the level of campaign spending, the interest groups need to know how their valence investments will affect the election probabilities of the candidates. To get the probabilities, we need to rearrange Equation 2 so it is in terms of \hat{X}_M . The left candidate

⁶We need only consider contributions from aligned interest groups. See Lemma A.1 in the Appendix.

will win the election if

$$\begin{aligned} -|x_L^* - \hat{x}_M| + v_L &> -|x_R^* - \hat{x}_M| + v_R \\ \hat{x}_M &< \frac{x_R^* + x_L^* - (v_R - v_L)}{2} \end{aligned}$$

We define $\bar{x}_M \equiv \frac{x_R^* + x_L^* - (v_R - v_L)}{2}$. Using this cutoff, we can state the interest group utilities.

We show the left group's utility, but the right group's is similar:

$$U_{G_L}(v_L) = \begin{cases} -|x_L^* - \hat{x}_{G_L}| - \frac{v_L^2}{4} & \text{if } \hat{x}_M < \frac{x_R^* + x_L^* - (v_R - v_L)}{2} \\ -\frac{1}{2}(|x_L^* - \hat{x}_{G_L}| + |x_R^* - \hat{x}_M|) - \frac{v_L^2}{4} & \text{if } \hat{x}_M = \frac{x_R^* + x_L^* - (v_R - v_L)}{2} \\ -|x_R^* - \hat{x}_{G_L}| - \frac{v_L^2}{4} & \text{if } \hat{x}_M > \frac{x_R^* + x_L^* - (v_R - v_L)}{2} \end{cases}$$

While the final policies are fixed, the probability $\hat{x}_M < \frac{x_R^* + x_L^* - (v_R - v_L)}{2}$ depends on how much both interest groups spends on the campaign. Given the fact that \hat{x}_M is distributed uniformly on, the probability that $\hat{x}_M < \bar{x}_M$ is $\frac{1 + \mu(x_R - x_L - (v_R - v_L))}{2}$. Similarly, the probability that $\hat{x}_M > \bar{x}_M$ is $\frac{1 - \mu(x_R - x_L - (v_R - v_L))}{2}$. Using this probabilities, we have these expected utilities:

$$\begin{aligned} EU_{G_L}(v_L) &= -|x_L^* - \hat{x}_{G_L}| \cdot \frac{1 + (x_R^* + x_L^* - (v_R - v_L))}{2} \\ &\quad - |x_R^* - \hat{x}_{G_L}| \cdot \frac{1 - (x_R^* + x_L^* - (v_R - v_L))}{2} - \frac{v_L^2}{4} \\ EU_{G_R}(v_R) &= -|x_L^* - \hat{x}_{G_R}| \cdot \frac{1 + (x_R^* + x_L^* - (v_R - v_L))}{2} \\ &\quad - |x_R^* - \hat{x}_{G_R}| \cdot \frac{1 - (x_R^* + x_L^* - (v_R - v_L))}{2} - \frac{v_R^2}{4} \end{aligned}$$

The key thing to note from these expected utilities is that the probability of the left policy portion of the expected utilities are increasing in v_L , and similarly the probability of the right policy portion is increasing in v_R . Each interest group will optimize over their candidates valence, which then converts into campaign spending. We give exact expressions

for equilibrium valance and spending in the next proposition.

Proposition 3.1. ⁷ *Equilibrium valence levels are as follows*

$$v_L^* = \begin{cases} (x_R^* - x_L^*) & \text{if } \hat{x}_{G_L} \leq \hat{x}_{P_L} \\ (x_R^* + x_L^* - 2\hat{x}_{G_L}) & \text{if } \hat{x}_{G_L} \geq \hat{x}_{P_L} \end{cases}$$

and

$$v_R^* = \begin{cases} (x_R^* - x_L^*) & \text{if } \hat{x}_{G_R} \geq \hat{x}_{P_R} \\ (2\hat{x}_{G_R} - x_R^* - x_L^*) & \text{if } \hat{x}_{G_R} \leq \hat{x}_{P_R} \end{cases}$$

Equilibrium campaign spending is $\frac{v_i^}{4}$ for all equilibrium valence levels.*

Valence investments (and therefore campaign spending) are equal when both interest groups are extreme. Valence investments are idiosyncratic when at least one of the interest groups is moderate. We will see later that the comparative statics are also different depending on whether the interest groups are extreme or moderate.

The valence invested is exactly the policy utility difference between the two policies. When an interest group is extreme, the difference in policy utility is simply the distance between the two final policies. Take the right group as an example:

$$\begin{aligned} & -|x_R^* - \hat{x}_{G_R}| - (-|x_L^* - \hat{x}_{G_R}|) \\ & = x_R^* - \hat{x}_{G_R} - (\hat{x}_{G_R} - x_L^*) \\ & = x_R^* - x_L^* \end{aligned}$$

However, if the right group is moderate (that is, if the group's ideal point is closer than

⁷All proofs are in the appendix

the right politician's ideal point to 0), we have

$$\begin{aligned}
& -|x_R^* - \hat{x}_{G_R}| - (-|x_L^* - \hat{x}_{G_R}|) \\
& = \hat{x}_{G_R} - x_R^* - (\hat{x}_{G_R} - x_L^*) \\
& = 2\hat{x}_{G_R} - x_R^* - x_L^*
\end{aligned}$$

The interest group's ideal point is between the two possible policies, and therefore the simple distance between policies would overstate the policy difference for the interest group. Crucially, note that the sign on x_R^* is different depending on whether the interest group is more or less extreme than the politician. As x_R^* increases, spending will increase with an extreme interest groups, but decrease with a moderate group.

The pattern is slightly different when looking at the interest group itself. As the interest group becomes more extreme, but still relatively moderate, spending increases. Once the interest groups becomes more extreme than the politician spending starts to slow down. When the interest groups is moderate, as it becomes more extreme there are two effects. One, it moves farther away from the opposing side's policy and two it moves closer to its own side's policy. Both of these effects work in the same direction, increasing spending.

An extreme interest groups that becomes more extreme still moves farther away from the opposing side's policy. However, now the group is also moving farther away from it's own side's policy. With post election lobbying, the interest is also pulling policy with it. Spending still increases, but at a slower rate than with a moderate interest group (if there is no lobbying, spending is flat once the interest groups is extreme). The relative alignment of interest groups and politician determines how quickly spending increases as groups become more extreme. As an example, 6 shows spending with no lobbying, with equal access lobbying (ie, $\alpha = 1$), and extra access lobbying ($\alpha = 1.5$). Note that this figure is just to give an illustration of the interest groups' possible behavior and how they respond to different access levels. As we would expected, greater access leads to more spending. The equilibrium as

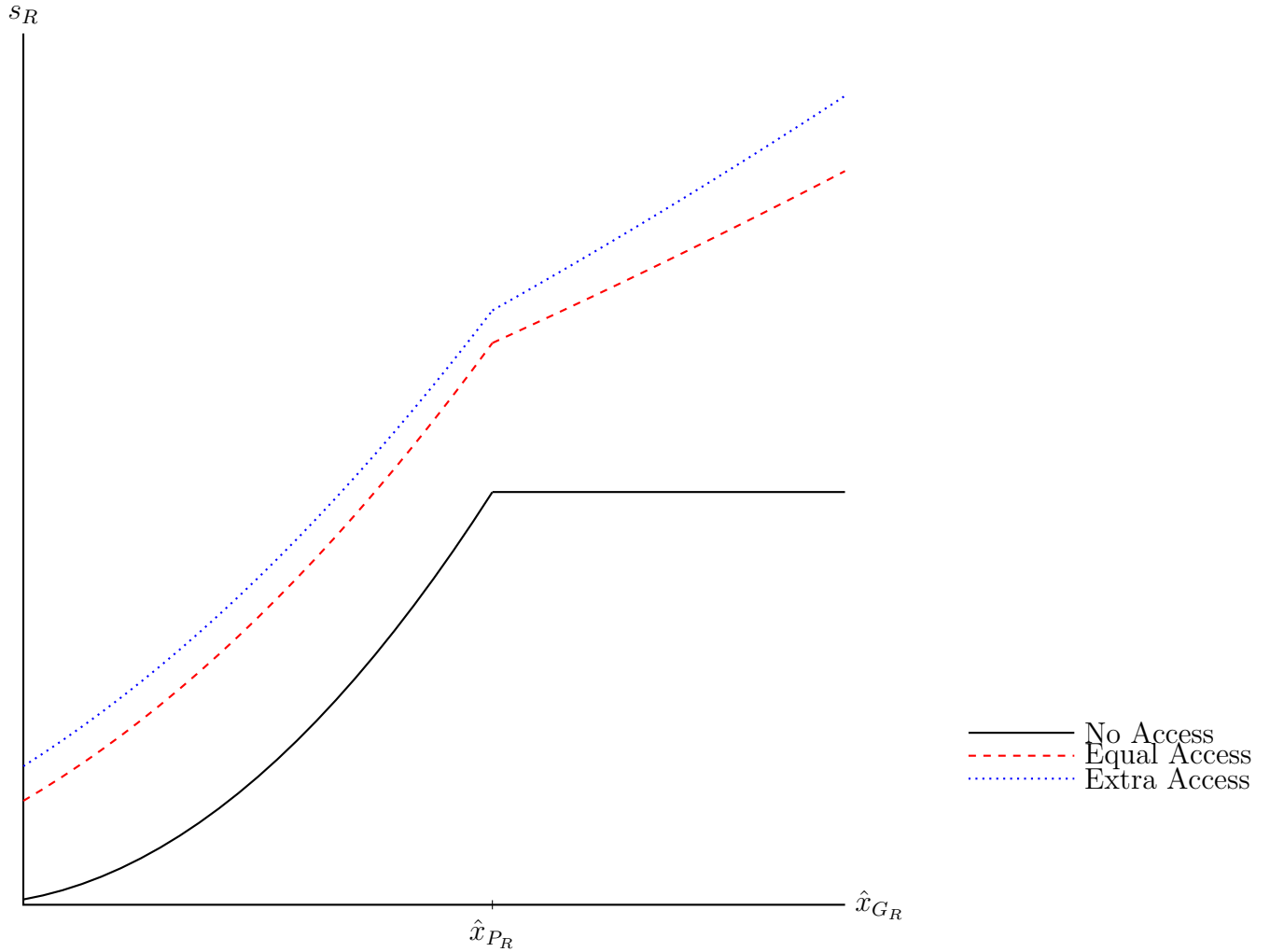


Figure 6: Expected Campaign Spending by Group Extremity

will be determined below.

3.3 Politician's Problem: Access

The politicians decide on lobbying access levels before the election. They must take into account not only the effect on their aligned interest group's investment, but also that increasing access may make the opposing interest group invest more. For example, say both interest groups are extreme. Then if the left politician increases the amount of access granted to the left politician, policy will be worse for the right interest group when the left politician wins. This makes winning the election more important for the right interest group, and they will

increase their own investment for the right candidate.

Substituting the optimal valence levels from the campaign stage into the politicians' expected utility functions yields the below expressions. Note that policies (x_L^* and x_R^*) as well as valence levels $v_R^* + v_L^*$ are functions of the access levels. The access choice affects the policy *payoff* part if the politician wins the election, but also affects the *probability* that she will win the election. The full expected utility functions are given below.

$$\begin{aligned}
EU_{P_L}(\alpha_L) &= -|x_L^* - \hat{x}_{P_L}| \cdot \frac{1 + (x_R^* + x_L^* - (v_R^* - v_L^*))}{2} \\
&\quad - |x_R^* - \hat{x}_{P_L}| \cdot \frac{1 - (x_R^* + x_L^* - (v_R^* - v_L^*))}{2} \\
EU_{P_R}(\alpha_R) &= -|x_L^* - \hat{x}_{P_R}| \cdot \frac{1 + (x_R^* + x_L^* - (v_R^* - v_L^*))}{2} \\
&\quad - |x_R^* - \hat{x}_{P_R}| \cdot \frac{1 - (x_R^* + x_L^* - (v_R^* - v_L^*))}{2}
\end{aligned}$$

The politicians optimize α_i over these expected utilities. The next proposition characterizes optimal access for all possible alignments:

Proposition 3.2. ⁸ *Equilibrium access levels are as follows*

$$(\alpha_L^*, \alpha_R^*) = \begin{cases} (0, 0) & \text{if } \hat{x}_{G_L} \leq \hat{x}_{P_L} \leq 0 \leq \hat{x}_{P_R} \leq \hat{x}_{G_R} \\ \left(0, \frac{4\hat{x}_{P_L} - 2\hat{x}_{G_L} - 1}{2(\hat{x}_{P_R} - \hat{x}_{G_R}) - (4\hat{x}_{P_L} - 2\hat{x}_{G_L} - 1)}\right) & \text{if } \hat{x}_{P_L} \leq \hat{x}_{G_L} \leq 0 \leq \hat{x}_{P_R} \leq \hat{x}_{G_R} \\ \left(\frac{-4\hat{x}_{P_R} + 2\hat{x}_{G_R} + 1}{2(\hat{x}_{G_L} - \hat{x}_{P_L}) - (-4\hat{x}_{P_R} + 2\hat{x}_{G_R} + 1)}, 0\right) & \text{if } \hat{x}_{G_L} \leq \hat{x}_{P_L} \leq 0 \leq \hat{x}_{G_R} \leq \hat{x}_{P_R} \\ (0, 0) & \text{if } \hat{x}_{P_L} \leq \hat{x}_{G_L} \leq 0 \leq \hat{x}_{G_R} \leq \hat{x}_{P_R} \end{cases}$$

The first thing to notice is that politicians only grant access when their interest group is extreme and the other interest groups is moderate. Why is this? First, when both interest

⁸For access, we consider only pure strategies. For some parameter values, no pure strategy equilibria exist. The following proposition characterizes the equilibrium access levels when equilibria in pure strategies exist. Conditions under which pure strategy equilibria exist are given in the appendix.

groups are extreme, there are no strictly positive levels of access in pure strategies. From Proposition 3.1, we know that when both interest groups are extreme, $v_L^* = v_R^*$. The two valence terms cancel each other out, and the voter makes her decision simply on the basis of politician ideal points.

The politicians will therefore have the same chance of winning the election whether they grant access to the interest groups or whether they grant no access. Granting positive access will only make policy worse for the politician without enhancing his electoral prospects. The interest groups will still contribute to the campaign because they still prefer their aligned politician's policy to the opposing policy. However, they do not also get access benefits.

Second, moderate groups respond less to access. Therefore the policy vs electoral success trade-off is not worth it when the interest groups are moderate. This leaves extreme aligned groups when opposing groups are moderate. These extreme groups invest enough in the campaign when given access to make it worthwhile to the politician and because the opposing group is moderate, this investment will be not canceled out like in the two extreme group case.

Before moving on, we can also show when the extreme interest group will grant the aligned interest group more say over the post-election policymaking process.

Corollary 3.2.1. *Let $\alpha_i^* > 0$. Then access is increasing in politician extremity and decreasing in interest group extremity.*

Essentially we should expect access to increase with political polarization, but decrease in interest group polarization.

4 Equilibrium Policies and Spending

Now that we have all of the choices in terms of model primitives, we can fully characterize final policies and campaign spending. We substitute the optimal access levels into the

expressions for policies and spending. We denote equilibrium values with the format

$$y_{i,w,z}^*$$

where

$$y \in \{x, v, s\}$$

$$i \in \{P_L, P_R, G_L, G_R\}$$

$$w, z \in \{m, e\}$$

x, v , and s stand for policy location, valence and spending while i denotes the player associated with the equilibrium value. m and e stand for moderate and extreme, with w representing the left interest group and z representing the right interest groups. For example,

$$x_{P_L m, e}^*$$

is the equilibrium policy when the left politician wins the election, the left interest group is moderate, and the right interest group is extreme. Similarly,

$$s_{G_R m, m}^*$$

is the equilibrium spending level for the right interest group when both interest groups are moderate. The next proposition states equilibrium policies.

Proposition 4.1. *Equilibrium policies are as follows*

$$(x_{P_L}^*, x_{P_R}^*) = \begin{cases} (\hat{x}_{P_L}, \hat{x}_{P_R}) & \text{if } \hat{x}_{G_L} \leq \hat{x}_{P_L} \leq 0 \leq \hat{x}_{P_R} \leq \hat{x}_{G_R} \\ (\hat{x}_{P_L}, \hat{x}_{P_R} - 2\hat{x}_{P_L} + \hat{x}_{G_L} + \frac{1}{2}) & \text{if } \hat{x}_{P_L} \leq \hat{x}_{G_L} \leq 0 \leq \hat{x}_{P_R} \leq \hat{x}_{G_R} \\ (\hat{x}_{P_L} - 2\hat{x}_{P_R} + \hat{x}_{G_R} - \frac{1}{2}, \hat{x}_{P_R}) & \text{if } \hat{x}_{G_L} \leq \hat{x}_{P_L} \leq 0 \leq \hat{x}_{G_R} \leq \hat{x}_{P_R} \\ (\hat{x}_{P_L}, \hat{x}_{P_R}) & \text{if } \hat{x}_{P_L} \leq \hat{x}_{G_L} \leq 0 \leq \hat{x}_{G_R} \leq \hat{x}_{P_R} \end{cases}$$

Two things to note before we move on to equilibrium spending. First, When both interest groups are extreme or when a group is moderate, final policies simply equal the politician's ideal point. This is because because equilibrium access is 0, and therefore the interest groups does not influence policy after the election.

Second, for a moderate-extreme interest group combination, the extreme interest group's ideal point does not appear in the expression for the final policy. This is because the combination of access and valence is calibrated to make sure the moderate interest groups does not over invest in response to the extreme group's campaign spending.

Proposition 4.2. *Equilibrium spending levels are as follows*

$$(s_{G_L}^*, s_{G_R}^*) = \begin{cases} \left(\frac{(\hat{x}_{P_R} - \hat{x}_{P_L})^2}{4}, \frac{(\hat{x}_{P_R} - \hat{x}_{P_L})^2}{4} \right) & \text{if } \hat{x}_{G_L} \leq \hat{x}_{P_L} \leq 0 \leq \hat{x}_{P_R} \leq \hat{x}_{G_R} \\ \left(\frac{(\frac{1}{2} - \hat{x}_{P_L} - \hat{x}_{G_L} + \hat{x}_{P_R})^2}{4}, \frac{(\frac{1}{2} - 3\hat{x}_{P_L} + \hat{x}_{G_L} + \hat{x}_{P_R})^2}{4} \right) & \text{if } \hat{x}_{P_L} \leq \hat{x}_{G_L} \leq 0 \leq \hat{x}_{P_R} \leq \hat{x}_{G_R} \\ \left(\frac{(\frac{1}{2} + 3\hat{x}_{P_R} - \hat{x}_{G_R} - \hat{x}_{P_L})^2}{4}, \frac{(\frac{1}{2} + \hat{x}_{P_R} + \hat{x}_{G_R} - \hat{x}_{P_L})^2}{4} \right) & \text{if } \hat{x}_{G_L} \leq \hat{x}_{P_L} \leq 0 \leq \hat{x}_{G_R} \leq \hat{x}_{P_R} \\ \left(\frac{(\hat{x}_{P_R} + \hat{x}_{P_L} - 2\hat{x}_{G_L})^2}{4}, \frac{(2\hat{x}_{G_R} - \hat{x}_{P_R} - \hat{x}_{P_L})^2}{4} \right) & \text{if } \hat{x}_{P_L} \leq \hat{x}_{G_L} \leq 0 \leq \hat{x}_{G_R} \leq \hat{x}_{P_R} \end{cases}$$

We first focus on the extreme-extreme and moderate-moderate combinations. Take the left interest group. From Proposition 4.1, we know that $x_{P_{L,e}}^* = x_{P_{L,m}}^*$. However, from Proposition 4.2, we know that $s_{G_{L,e}}^* \neq s_{G_{L,m}}^*$. This suggests that policies themselves are not sufficient to understand why a certain amount was spent on a campaign.

For the moderate-extreme alignments, observe that the just like the equilibrium policies, the moderate interest group's ideal point enters the expression for equilibrium spending.

Corollary 4.2.1. *In moderate-extreme or extreme-moderate alignments, the extreme interest groups always spends more than the moderate group.*

Corollary 4.2.1 is due to two facts. One, the extreme interest group was granted access, and this makes electing their aligned candidate more appealing. Two, the moderate interest groups is between the two possible policies. This means that the policy loss of the opposing policy vs the aligned policy must be less than the policy loss for the extreme interest group. Therefore extreme interest group is willing to spend more than the moderate group to get their candidate elected.

There are a few comparative statics worth highlighting. Again, the relative alignment will be key. First we look at changing politician ideal points under extreme-extreme and moderate-moderate alignments.

Proposition 4.3. *Let both interest groups be extreme. Then spending increases (decreases) for both interest groups as either politician becomes more extreme (moderate).*

Let both interest groups be moderate. Then spending increases (decreases) for both groups when the opposing politician becomes more extreme (moderate) and decreases (increases) when the aligned politician becomes more extreme (moderate).

When both interest groups are extreme, an increase in political polarization (that is, an increase in the difference between the two politician's ideal points) increases the stakes of the election for the interest groups. The difference between the two possible policies is greater, and this leads the interest groups to spend more money on they election. Note that it does not matter how political polarization increases; an increase due to the aligned or opposing politician has the same effect.

When the interest groups are moderate, however, the aligned politician has a different effect. An increase in the extremity of the opposing politician makes the difference in policy

utility greater, and therefore leads to more investment as in the extreme-extreme case. However, an increase in the aligned politician's extremity makes the difference in policy *smaller* for the aligned politician. A more extreme politician moves farther away from a moderate interest group, and they correspondingly decrease their investments. The next proposition on comparative statics for the moderate-extreme alignments further shows how important alignment is to understanding campaign spending.

Proposition 4.4. *Let one interest group be moderate and one interest group be extreme. Then both interest groups increase spending when either politician becomes more extreme.*

The extreme interest group decreases spending when the moderate interest group becomes more extreme, but the moderate interest group increases spending when it becomes more extreme.

Recall from Corollary 3.2.1 that politician polarization increases access but interest group polarization decreases access. We can see how this affects spending in Proposition 4.4. For example, let the left interest group be moderate and the right interest group be extreme. Then if the left politician becomes more extreme, both groups increase spending. For the left group, the left politician is actually moving farther away from \hat{x}_{G_L} , which pushes the group to decrease spending. However, this is counteracted by the increase in access granted to the right group. This increased access pushes the right policy away from \hat{x}_{G_L} by far enough to increase the left group's spending.

Figures 7 and 8 summarize how spending changes as players become more extreme. Figure 7 in particular highlights the importance of considering post-election lobbying. Without lobbying, the right group's spending pattern would be the same regardless of whether the left interest was moderate or extreme. When the left group can lobby, however, the right group's spending keeps increasing even as the right politician becomes even more extreme.

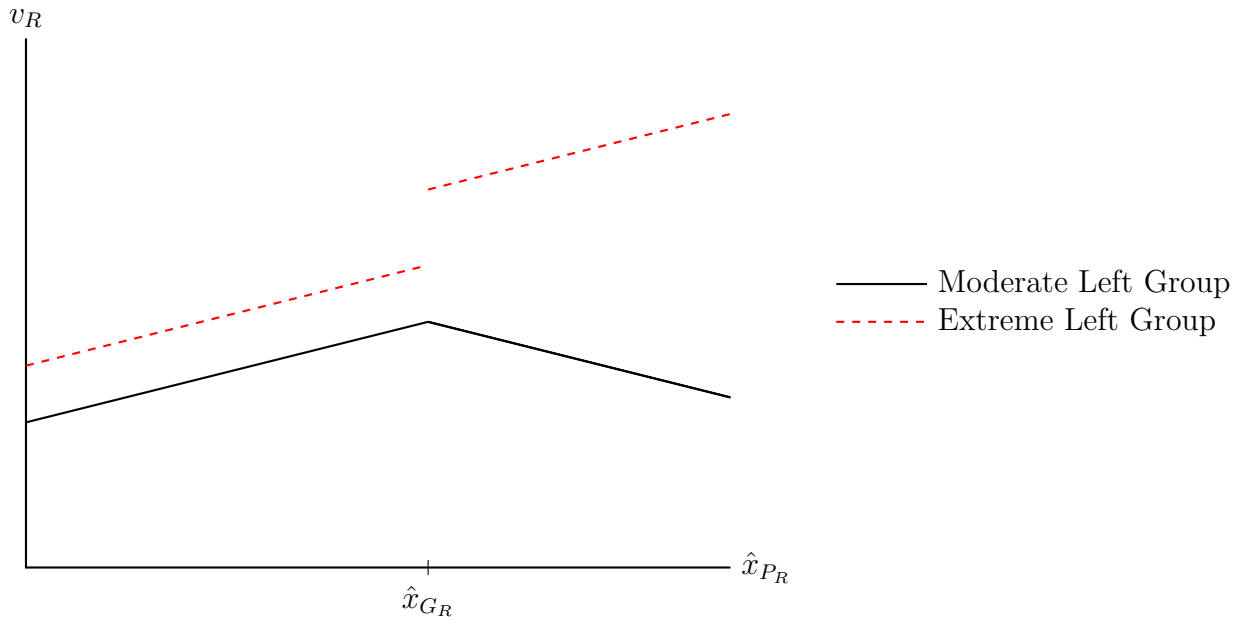


Figure 7: Expected Campaign Spending by Politician Extremity

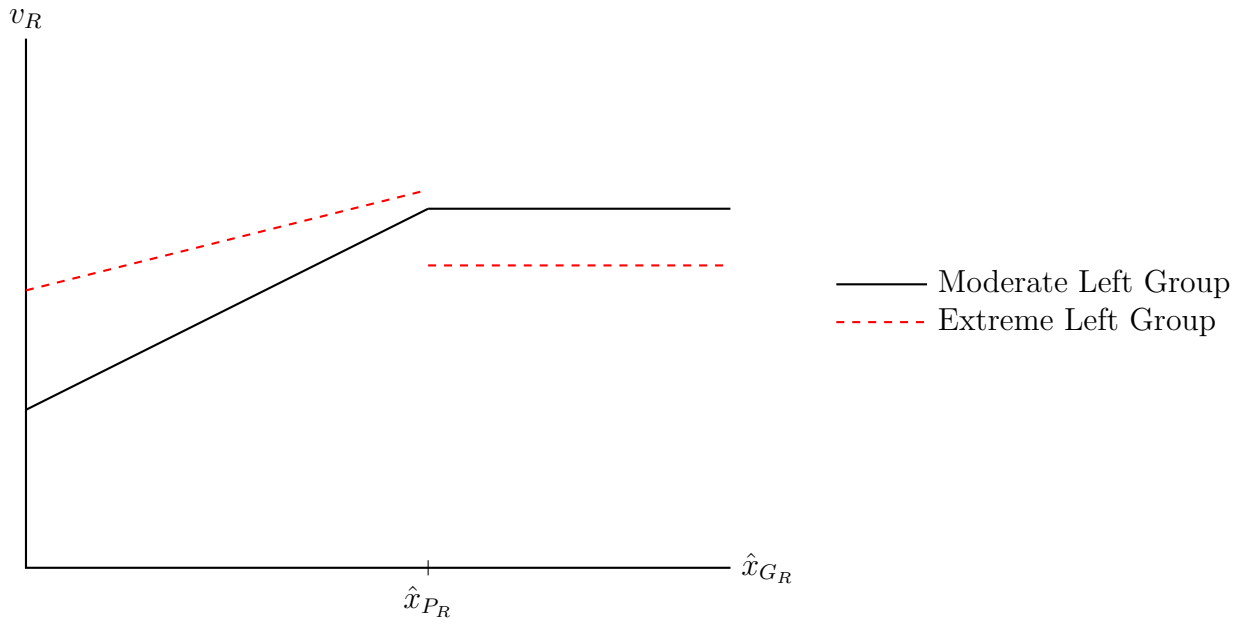


Figure 8: Expected Campaign Spending by Group Extremity

4.1 Spending and Electoral Success

Politicians and interest groups that spend a lot on an election are often accused of trying to buy the election (Fuller, 2014). In this section we see how campaign spending correlated with the probability of winning the election. First, note that the more moderate the policy, the greater the chance the politician has of winning the election. Therefore the following result is equivalent to one involving policy moderation instead and election probability.

Lemma 4.1. *The probability of winning the election is*

1. *decreasing in aligned politician extremity*
2. *increasing in aligned interest group extremity*
3. *increasing in opposing politician extremity*
4. *decreasing in opposing interest group extremity*

We can then use this lemma to show how spending correlated with the chances of winning the election.

Proposition 4.5. *Let both interest groups be moderate. Then spending and the winning probability are both decreasing in politician extremity. Otherwise, the electoral probability is decreasing in politician extremity but spending is increasing.*

Figures 9 and 10 illustrate this proposition nicely. When both groups are moderate, and increase in the aligned politician's extremity makes the utility difference between the two possible outcomes smaller. Therefore the interest group is less willing to spend money and the more extreme politician is more likely to lose the election. In other alignments, an increase in the aligned politician's extremity increases the utility differences between the two possible outcomes. This increases spending to try and help the politician's electoral chances.

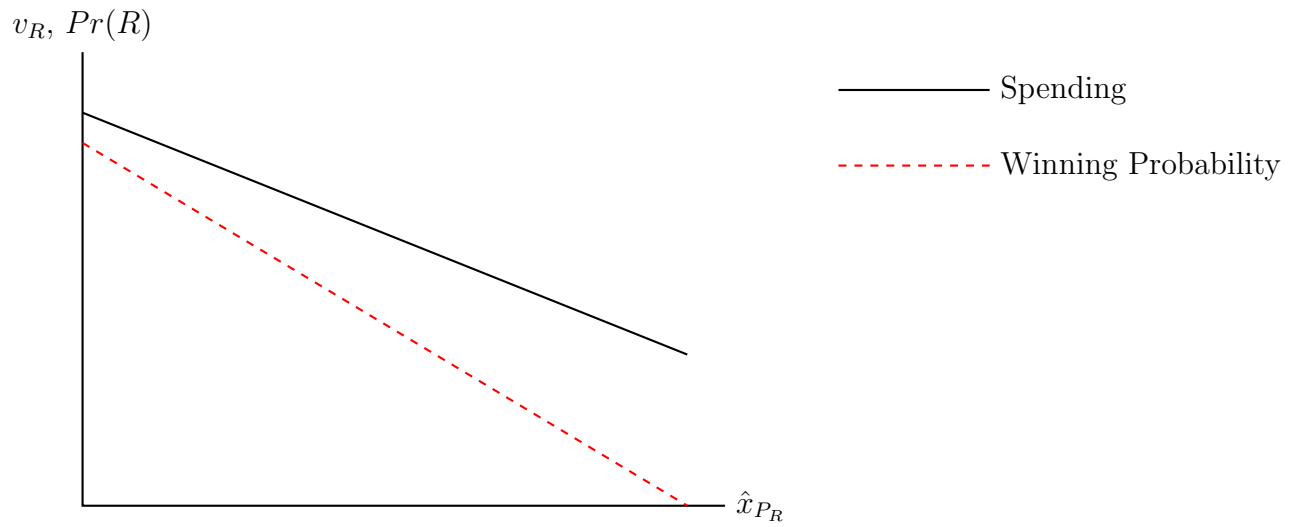


Figure 9: Spending and Winning Probability for the Right Group with Two Moderate Groups

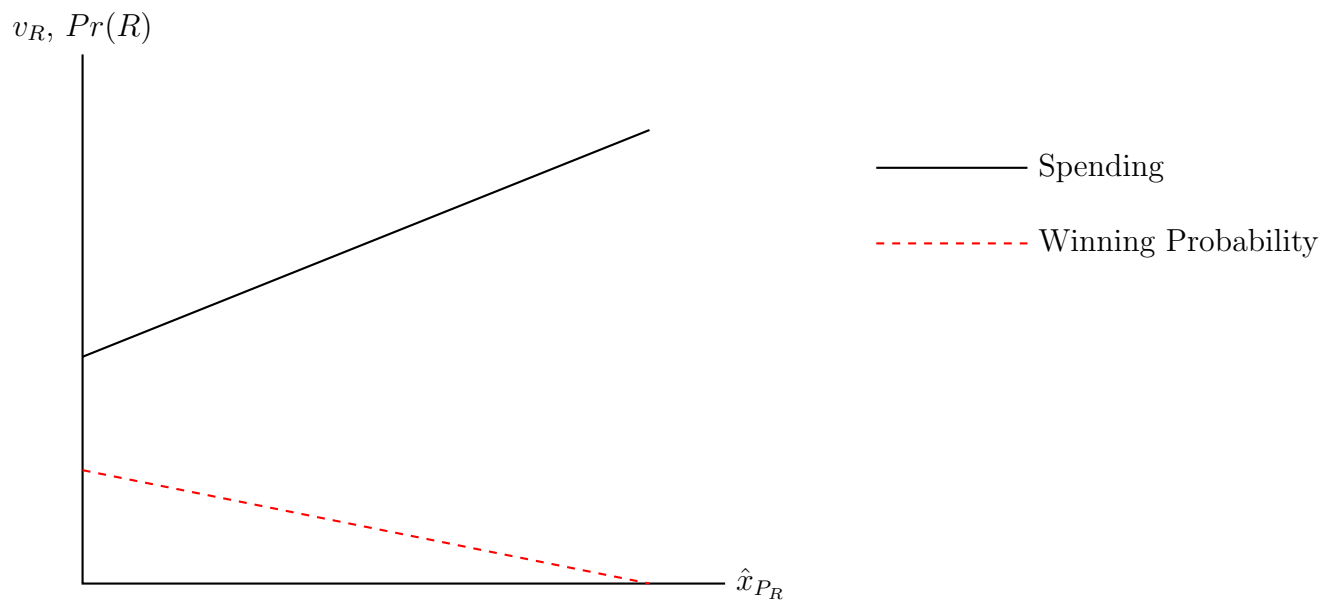


Figure 10: Spending and Winning Probability for the Right Group with Two Extreme Groups

5 Conclusion

In this paper we have analyzed an electoral model with access, campaign spending, and lobbying. We want to emphasize that the alignment of politicians and interest groups interacts with the polarization of politicians and interest groups to drive spending. We conclude with a few thoughts on empirical implications and possibility for future research.

5.1 Empirical Implications

The comparative statics also have important implications for empirical research. First, we show how spending is conditional on the *relative* polarization between interest groups and politicians. Therefore studies that look only at politician ideology or just at interest group ideology may have an omitted variable problem. Researchers must consider interest group and politician ideology jointly to understand how they affect campaign spending. For instance, previous studies such as McKay (2010) and Thieme (2019) show that more extreme interest groups contribute more to campaigns. While our model predicts such effects, it comes with the additional prediction that this relationship may attenuate once groups are more polarized than politicians. When groups are more polarized than politicians, further group polarization entails less alignment with the group's preferred politician.

5.2 Future Work

A number of potentially fruitful avenues for related work presented themselves throughout our analysis. Future work may wish to extend the model to speak to different limitations on spending. For example, how would a limit on campaign expenditures affect the probability each candidate wins the election? In our model, this should have no effect when both interest groups are extreme since these investments cancel each other out. However, this may attenuate the influence of an extreme group when the other group is moderate.

We did not allow politicians to make campaign investments in their own campaigns,

instead focusing on interest group decision making. Certainly this comes at a loss of some verisimilitude. How does interest group spending and lobbying change when politicians adopt a strategic role in the campaign stage?

A final suggestion is to consider multiple political offices. Large interest groups give to multiple races at once. How does future lobbying of coalitions of politicians affect the distribution of spending across many simultaneous campaigns? The strategic interrelation of campaigns and lobbying points towards many avenues of interesting and worthwhile future research. This list comprises but a small sample of these.

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A Proofs

Lemma A.1. *Interest groups only contribute to their aligned candidate.*

Proof. Contributions increase the election probability of a candidate. Giving contributions to the opposing candidate would make it more likely the opposing policy would be enacted. Therefore the interest would be lowering its policy utility at a cost. This could never be optimal. ■

Lemma A.2. *The probability that $x_M > \bar{x}_M$ is $\frac{1-\mu(x_R-x_L-(v_R-v_L))}{2}$ and the probability that $x_M < \bar{x}_M$ is $\frac{1+\mu(x_R-x_L-(v_R-v_L))}{2}$.*

Proof. Because \hat{x}_M is distributed uniformly on $[-\frac{1}{2}, \frac{1}{2}]$, the probability that $\hat{x}_M > \bar{x}_M$

$$\frac{\frac{1}{2} - \hat{x}_M}{\frac{1}{2} - \frac{x_{P_R}^* + x_{P_L}^* - (v_R - v_L)}{2}} = \frac{1 - (x_{P_R}^* + x_{P_L}^* - (v_R - v_L))}{2}$$

And the probability that $x_M < \bar{x}_M$ is

$$1 - \frac{1 - (x_{P_R}^* + x_{P_L}^* - (v_R - v_L))}{2} = \frac{1 + (x_{P_R}^* + x_{P_L}^* - (v_R - v_L))}{2}$$

■

Proof of Proposition 3.1. We show this for v_L, v_R is similar.

The expected utility for the left group is:

$$\begin{aligned}
EU_{G_L}(v_L) &= -|x_{P_L}^* - \hat{x}_{G_L}| \cdot Pr(x_M < \bar{x}_M) - |x_{P_R}^* - \hat{x}_{G_L}| \cdot Pr(x_M > \bar{x}_M) - \frac{v_L^2}{4} \\
&= -|x_{P_L}^* - \hat{x}_{G_L}| \cdot \frac{1 + (x_{P_R}^* + x_{P_L}^* - (v_R - v_L))}{2} \\
&\quad - |x_{P_R}^* - \hat{x}_{G_L}| \cdot \frac{1 - (x_{P_R}^* + x_{P_L}^* - (v_R - v_L))}{2} - \frac{v_L^2}{4}
\end{aligned}$$

Maximizing with respect to v_L , we get this first order condition when $\hat{x}_{G_L} < x_{P_L}^*$:

$$\frac{\partial EU_{G_L}(v_L)}{\partial v_L} = -\frac{(x_{P_L}^* - \hat{x}_{G_L})}{2} + \frac{(x_{P_R}^* - \hat{x}_{G_L})}{2} - \frac{v_L}{2}$$

Setting this equal to 0,

$$v_L^* = (x_{P_R}^* - x_{P_L}^*)$$

Similarly,

$$v_R^* = x_{P_R}^* - x_{P_L}^*$$

If $\hat{x}_{G_L} > x_{P_L}^*$, the first order condition is

$$\frac{\partial EU_{G_L}(v_L)}{\partial v_L} = \frac{(x_{P_L}^* - \hat{x}_{G_L})}{2} + \frac{(x_{P_R}^* - \hat{x}_{G_L})}{2} - \frac{v_L}{2}$$

Setting this equal to 0,

$$v_L^* = x_{P_R}^* + x_{P_L}^* - 2\hat{x}_{G_L}$$

For G_R , we have

$$v_R^* = 2\hat{x}_{G_R} - x_{P_R}^* - x_{P_L}^*$$

■

Proof of Proposition 3.2. We outline the procedure used to find the optimal levels of access.

See the mathematica file for the computational details.

First, maximize their expected utility over α_i :

$$\begin{aligned} EU_{P_L}(\alpha_L) &= -|x_L^* - \hat{x}_{P_L}| \cdot \frac{1 + (x_R^* + x_L^* - (v_R^* - v_L^*))}{2} \\ &\quad - |x_R^* - \hat{x}_{G_L}| \cdot \frac{1 - (x_R^* + x_L^* - (v_R^* - v_L^*))}{2} \\ EU_{P_R}(\alpha_R) &= -|x_L^* - \hat{x}_{P_R}| \cdot \frac{1 + (x_R^* + x_L^* - (v_R^* - v_L^*))}{2} \\ &\quad - |x_R^* - \hat{x}_{G_R}| \cdot \frac{1 - (x_R^* + x_L^* - (v_R^* - v_L^*))}{2} \end{aligned}$$

Call the solutions to this problem $\tilde{\alpha}_L$ and $\tilde{\alpha}_R$.

Second, solve the system of two equations to find α_L^* and α_R^* . Check to see that these optimal α s are greater than 0. If they are not, substitute 0 into $\tilde{\alpha}_L$ and $\tilde{\alpha}_R$. Check to see if $\tilde{\alpha}_L(0)$ and $\tilde{\alpha}_R(0)$ are greater than 0. If neither are, $\alpha_L^* = \alpha_R^* = 0$.

If one is greater than 0, but the other is still negative (say $\tilde{\alpha}_L(0) > 0$ and $\tilde{\alpha}_R(0) < 0$), then $\alpha_L^* = \tilde{\alpha}_L(0)$ and $\alpha_R^* = 0$.

This procedure gives us

$$(\alpha_L^*, \alpha_R^*) = \begin{cases} (0, 0) & \text{if } \hat{x}_{G_L} \leq \hat{x}_{P_L} \leq 0 \leq \hat{x}_{P_R} \leq \hat{x}_{G_R} \\ \left(0, \frac{4\hat{x}_{P_L} - 2\hat{x}_{G_L} - 1}{2(\hat{x}_{P_R} - \hat{x}_{G_R}) - (4\hat{x}_{P_L} - 2\hat{x}_{G_L} - 1)}\right) & \text{if } \hat{x}_{G_L} \geq \hat{x}_{P_L} \leq \hat{x}_{G_L} \leq 0 \leq \hat{x}_{P_R} \leq \hat{x}_{G_R} \\ \left(\frac{-4\hat{x}_{P_R} + 2\hat{x}_{G_R} + 1}{2(\hat{x}_{G_L} - \hat{x}_{P_L}) - (-4\hat{x}_{P_R} + 2\hat{x}_{G_R} + 1)}, 0\right) & \text{if } \hat{x}_{G_L} \geq \hat{x}_{P_L} \leq \hat{x}_{P_L} \leq 0 \leq \hat{x}_{G_R} \leq \hat{x}_{P_R} \\ (0, 0) & \text{if } \hat{x}_{P_L} \geq \hat{x}_{G_L} \leq \hat{x}_{P_L} \leq 0 \leq \hat{x}_{G_R} \leq \hat{x}_{P_R} \end{cases}$$

■

Proof of Corollary 3.2.1. We show this for α_L^* , is the same for α_R^* .

$$\alpha_L^* = \frac{-4\hat{x}_{P_R} + 2\hat{x}_{G_R} + 1}{2(\hat{x}_{G_L} - \hat{x}_{P_L}) - (-4\hat{x}_{P_R} + 2\hat{x}_{G_R} + 1)}$$

$$\begin{aligned} \frac{\partial \alpha_L^*}{\partial \hat{x}_{P_L}} &= \frac{2(2\hat{x}_{G_R} - 4\hat{x}_{P_R} - 1)}{(1 + 2\hat{x}_{G_L} - 2\hat{x}_{G_R} - 2\hat{x}_{P_L} + 4\hat{x}_{P_R})^2} \leq 0 \\ \frac{\partial \alpha_L^*}{\partial \hat{x}_{P_R}} &= \frac{8(\hat{x}_{P_L} - \hat{x}_{G_L})}{(1 + 2\hat{x}_{G_L} - 2\hat{x}_{G_R} - 2\hat{x}_{P_L} + 4\hat{x}_{P_R})^2} \geq 0 \\ \frac{\partial \alpha_L^*}{\partial \hat{x}_{G_L}} &= \frac{2(-2\hat{x}_{G_R} + 4\hat{x}_{P_R} + 1)}{(1 + 2\hat{x}_{G_L} - 2\hat{x}_{G_R} - 2\hat{x}_{P_L} + 4\hat{x}_{P_R})^2} \geq 0 \\ \frac{\partial \alpha_L^*}{\partial \hat{x}_{G_R}} &= \frac{4(\hat{x}_{G_L} - \hat{x}_{P_L})}{(1 + 2\hat{x}_{G_L} - 2\hat{x}_{G_R} - 2\hat{x}_{P_L} + 4\hat{x}_{P_R})^2} \leq 0 \end{aligned}$$

■

Proof of Proposition 4.1. We simply substitute optimal access levels from Proposition 3.2 into the formulas for final policies, $\frac{\alpha_L^* \hat{x}_{G_L} + \hat{x}_{P_L}}{\alpha_L^* + 1}$ and $\frac{\alpha_R^* \hat{x}_{G_j} + \hat{x}_{P_j}}{\alpha_R^* + 1}$, and obtain the required results.

■

Proof of Proposition 4.2. We substitute the policies from Proposition 4.1 into the expressions from Proposition 3.1 to obtain equilibrium valence levels. Then, we square them and divide by 4.

■

Proof of Corollary 4.2.1. We show this for an extreme left group and a moderate right group. The opposite case is the same. Also note that if the valence is higher, the spending is higher.

$$v_L^* = \frac{1}{2} + 3\hat{x}_{P_R} - \hat{x}_{G_R} - \hat{x}_{P_L}, \quad v_R^* = \frac{1}{2} + \hat{x}_{P_R} + \hat{x}_{G_R} - \hat{x}_{P_L}.$$

Note that because the right interest groups is moderate, $\hat{x}_{P_R} \geq \hat{x}_{G_R}$. Therefore $3\hat{x}_{P_R} - \hat{x}_{G_R} \geq \hat{x}_{P_R} + \hat{x}_{G_R}$.

■

Proof of Propositions 4.3 and 4.4. We show this with respect to the valence levels since they are simpler expressions than the spending levels but move in the same direction. We take

the valence levels from Proposition 4.2 and then take derivatives with respect to the relevant parameters. See the mathematica notebook for the details. ■

Proof of Lemma 4.1. We substitute the valence levels from Proposition 4.2 into the probabilities from Lemma A.2 and then take derivatives with respect to the relevant parameters. See the mathematica notebook for the details. ■

[Proof of Proposition 4.5.] A straightforward comparison of the comparative statics from Propositions 4.3 and 4.4 and Lemma 4.1 provide the desired result. ■